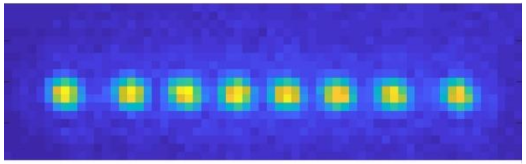
Quantum Science Seminar. April 2020

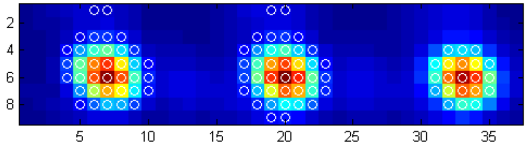
Trapped-ion quantum computing: A Coherent Control Problem

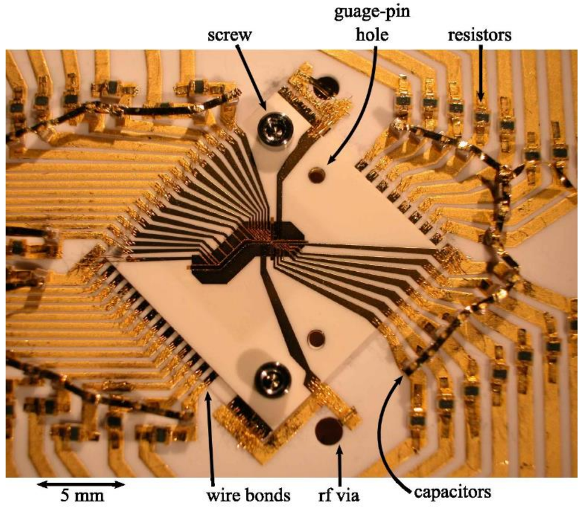


Roee Ozeri

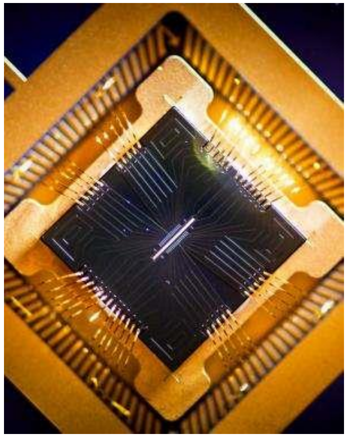
Weizmann Institute of Science

Laser-cooled Crystals of trapped atomic ions





Trap by NIST



Trap made by Sandia

$|1\rangle$

$|0\rangle$

Separation of few μm

Temperature few μK

Interface with qubit using lasers or microwaves

Qubit encoded in internal levels of ion

Trapped ion interaction with Radiation

Coupling between the two qubit levels using EM radiation.

For a single ion: $H(t) = H_0 + V(t)$

$H_0 = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \hbar\omega_m(\hat{a}^\dagger\hat{a} + \frac{1}{2})$ and $V(t) = \hbar\Omega_0(\hat{\sigma}^+ + \hat{\sigma}^-)cos(\mathbf{k}\hat{x} - \omega t + \phi)$

Where: $\hat{\sigma}_+ = |\uparrow\rangle\langle\downarrow|$; $\hat{\sigma}_- = |\downarrow\rangle\langle\uparrow|$

$$k\hat{x} = kx_{eq} + kx_0(\hat{a}^\dagger + \hat{a}) \equiv kx_{eq} + \eta(\hat{a}^\dagger + \hat{a}) \qquad x_0 = \sqrt{\frac{\hbar}{2M\omega_m}}$$

Trapped ion interaction with Radiation

In the interaction representation and within the Rotating Wave Appr. (RWA)

$$H_{int}(t) = \hbar\Omega_0/2\hat{\sigma}_+ \exp(i\eta(\hat{a}e^{-i\omega_m t} + \hat{a}^\dagger e^{i\omega_m t}))e^{i(kx_{eq}+\phi-\delta t)} + H.C.$$

When $\delta = s\omega_m$, only $|\downarrow, n\rangle$ and $|\uparrow, n+s\rangle$ will be resonantly coupled (another RWA).

Carrier: $s = 0$

$$\hat{H}_{int} = \frac{\hbar\Omega_0}{2} D_{n,n}(\hat{\sigma}_+ e^{i\phi} + \hat{\sigma}_- e^{-i\phi})$$

Red sideband (RSB): $s = -1$

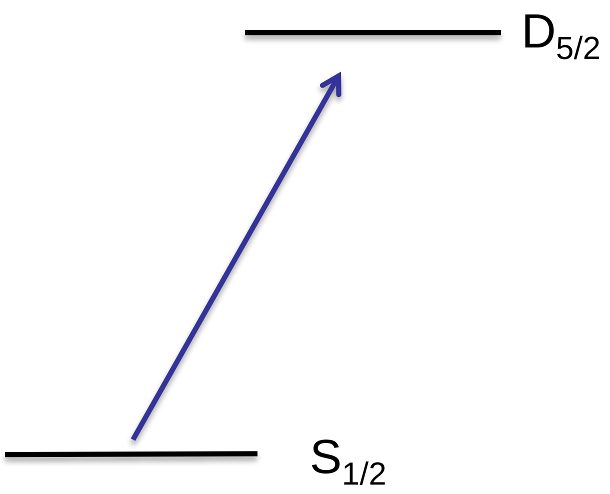
$$\hat{H}_{int} = \frac{\hbar\Omega_0}{2} D_{n-1,n}(\hat{a}\hat{\sigma}_+ e^{i\phi} + \hat{a}^\dagger\hat{\sigma}_- e^{-i\phi})$$

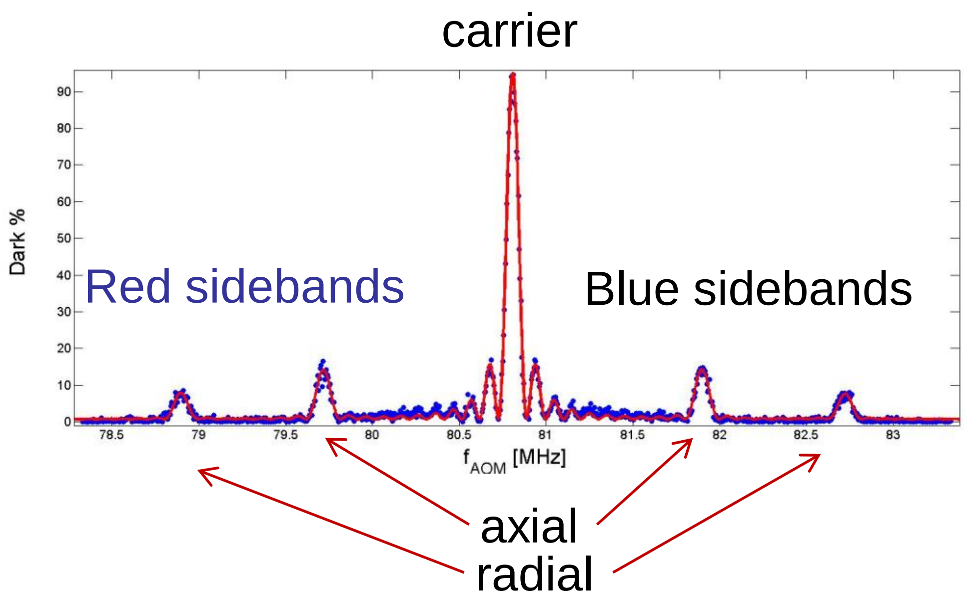
Blue sideband (BSB): $s = +1$

$$\hat{H}_{int} = \frac{\hbar\Omega_0}{2} D_{n+1,n}(\hat{a}^\dagger\hat{\sigma}_+ e^{i\phi} + \hat{a}\hat{\sigma}_- e^{-i\phi})$$

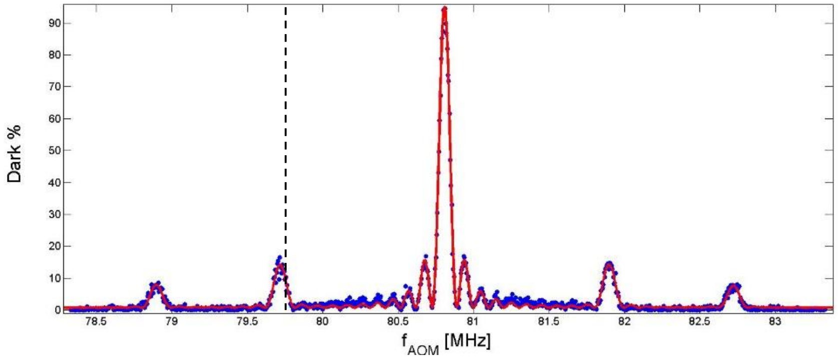
Sideband Spectroscopy

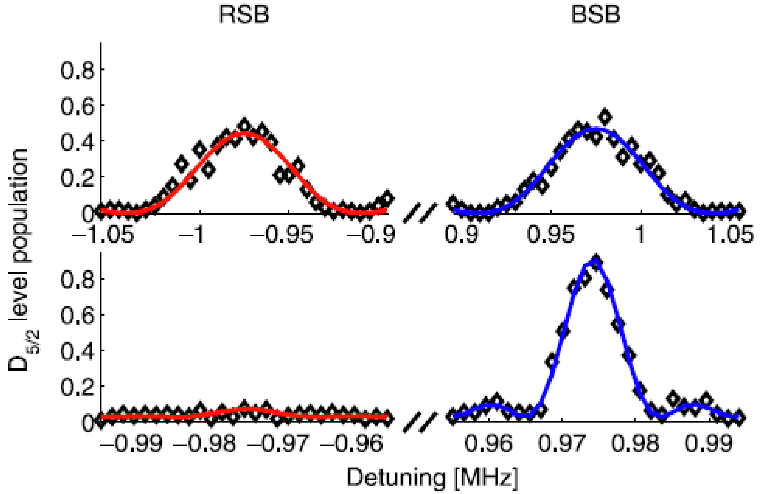
- Scan the laser frequency across the S →D transition





Sideband cooling to the ground state

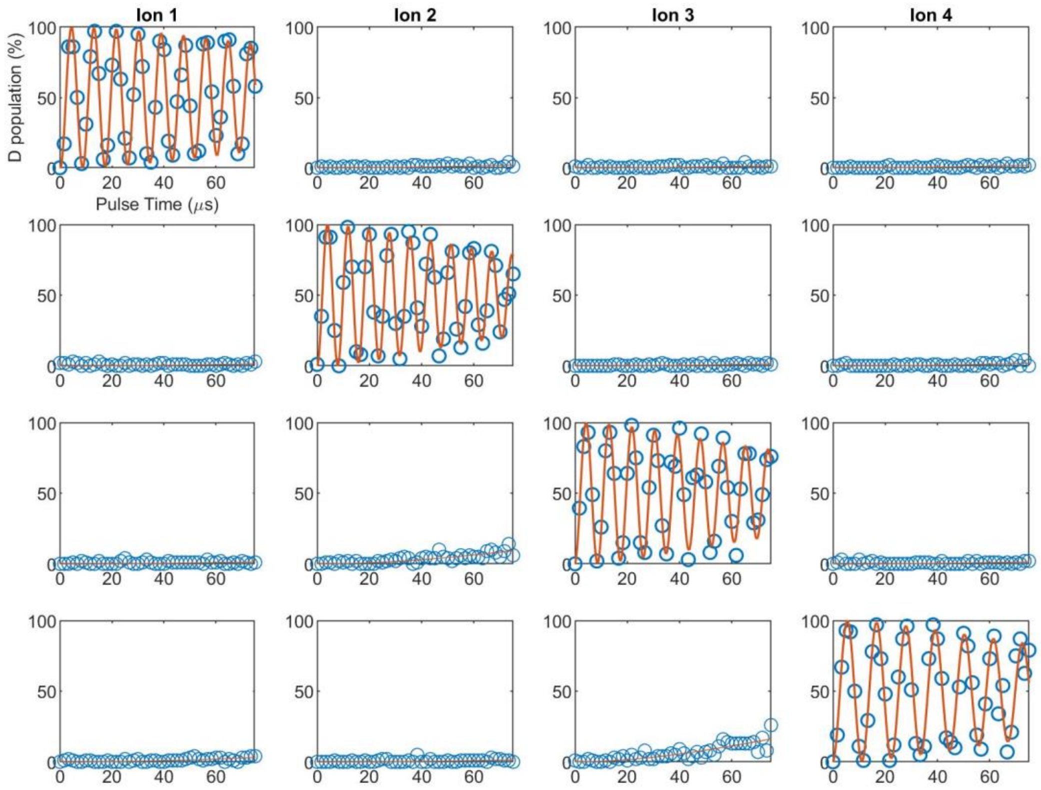




Pulse on red sideband together with optical-pumping

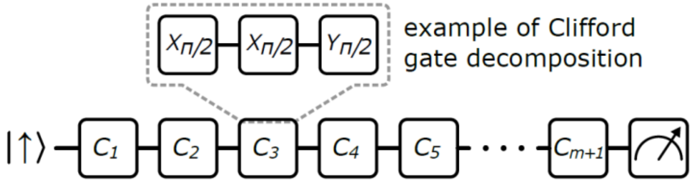
$\langle n \rangle < 0.05$ T $\approx$ 2 μK

Single qubit gates – Carrier rotations

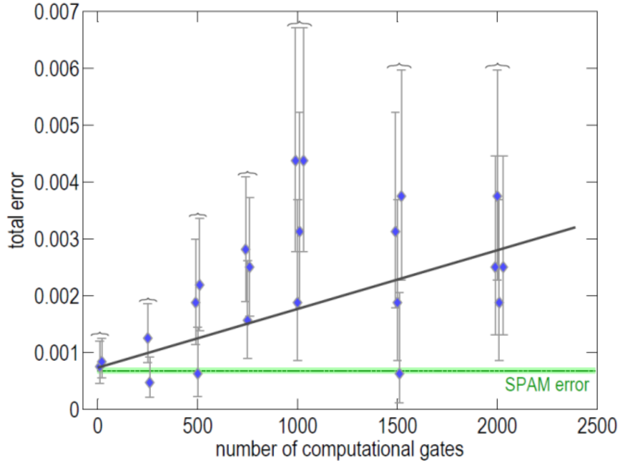


Tom Manovitz; MSc thesis (2016)

Randomized benchmarking

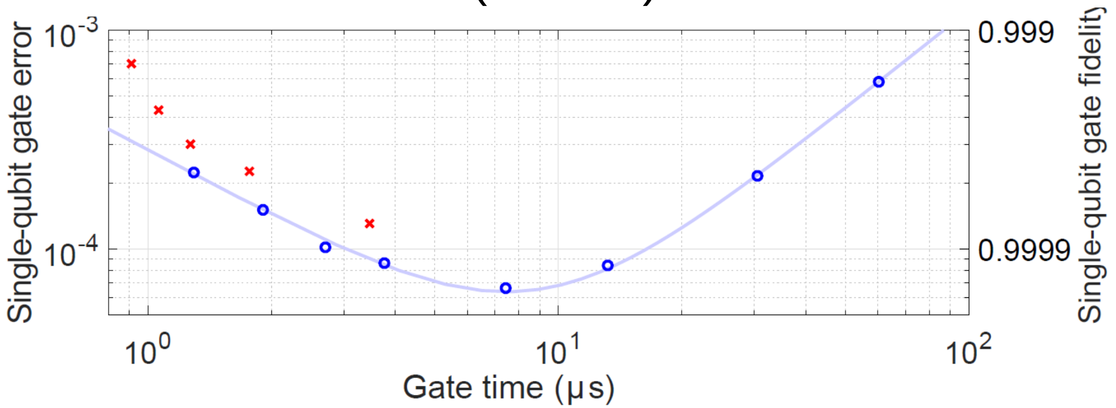


microwave



$\varepsilon \cong 10^{-6}$

Laser (Raman)



$\varepsilon \cong 10^{-4}$

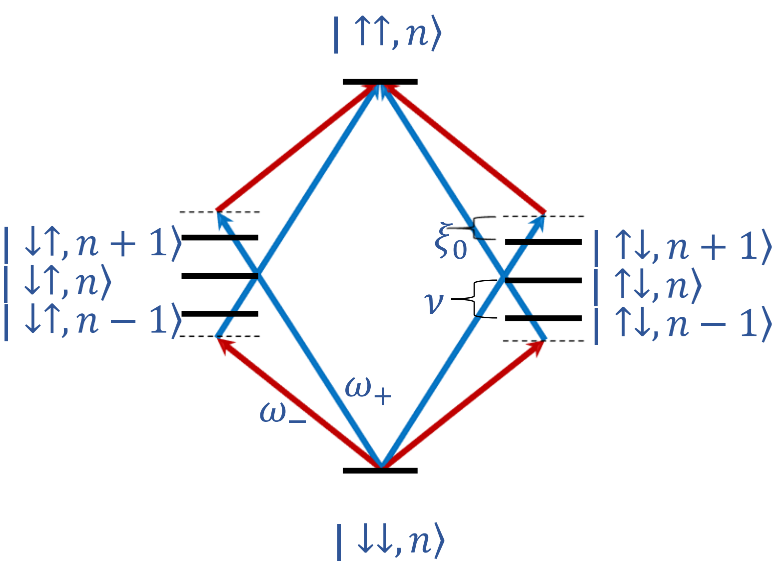
Lucas, Oxford

Entanglement in trapped ion systems

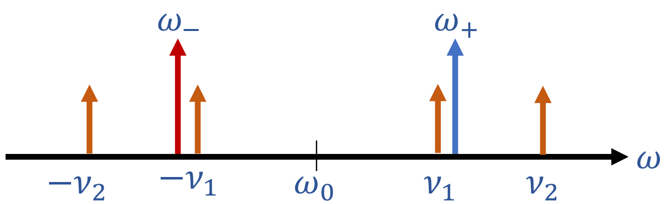
Mølmer-Sørensen gates

Use a bi-chromatic drive:

$$\omega_{\pm} = \omega_0 \pm (\nu + \xi_0)$$



Spectrally:



Hamiltonian (appx.) – spin dependent force:

$$H = \hbar\Omega \hat{J}_y \left(f(t)\hat{x} + g(t)\hat{p}\right)$$

Global spin operator:

$$\hat{J}_y = \frac{\hat{\sigma}_1^y + \hat{\sigma}_2^y}{2}$$

Harmonic oscillator operators

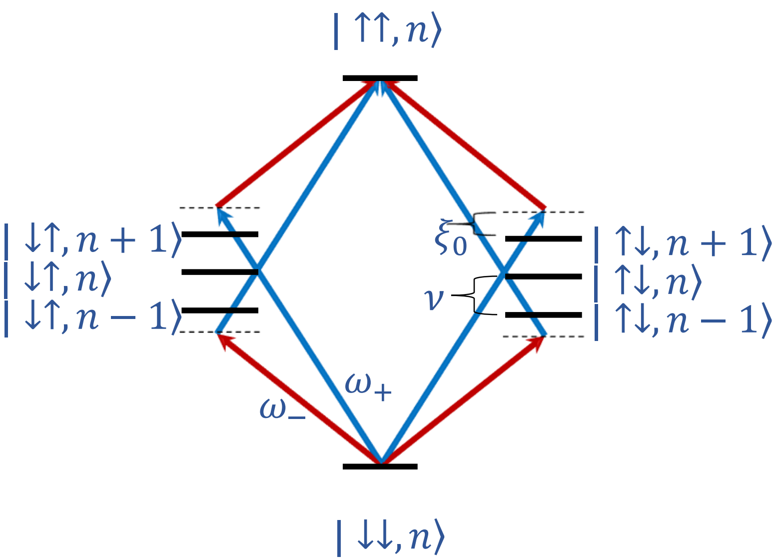
A. Sørensen & K. Mølmer, PRL (1999) & PRA (2000)

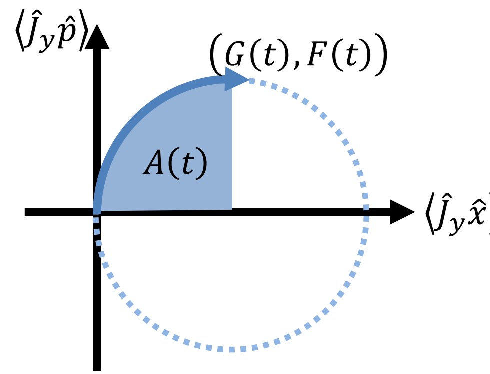
Mølmer – Sørensen gate

Bi-chromatic drive:

$$\omega_{\pm} = \omega_{SD} \pm (\nu + \boldsymbol{\xi_0})$$

$$\hat{U}(t;0) = e^{-iA(t)\frac{\hat{\sigma}_y \otimes \hat{\sigma}_y}{2}} e^{-iF(t)\hat{J}_y \hat{x}} e^{-iG(t)\hat{J}_y \hat{p}}$$





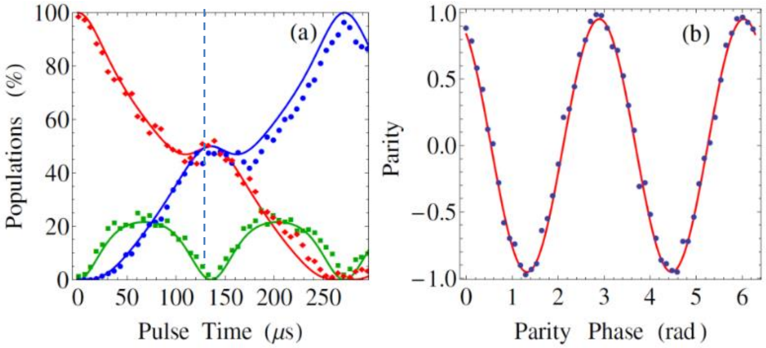
MS gate: $A(T) = \pi/2\,,\; F(T) = G(T) = 0$

$$|\downarrow\downarrow\rangle \rightarrow \boxed{\hat{U}_{MS}(T;0)} \rightarrow \frac{|\downarrow\downarrow\rangle - i|\uparrow\uparrow\rangle}{\sqrt{2}}$$

A. Sørensen & K. Mølmer, PRL (1999) & PRA (2000)

Laser-driven entanglement

SS; DD; SD + DS



$$\hat{U}_{MS}(T;0)|SS\rangle = \frac{|SS\rangle - i|DD\rangle}{\sqrt{2}}$$

Great, but sensitive to errors in gate parameters:
Time, trap frequency, laser detuning, laser intensity

Benhelm *et. al. Nat. Phys.* ***4*** 463 (2008)

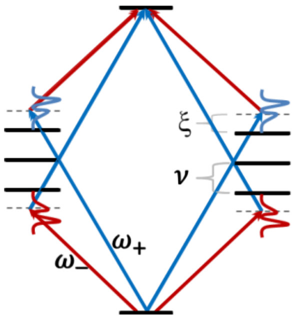
Akerman, Navon, Kotler, Glickman, RO, NJP 17, 113060 (2015)

Quantum Engineering of Robust Gates

Multi-tone Mølmer – Sørensen gates

$$\omega_{\pm,i} = \omega_0 \pm (\nu + n_i \xi_0)$$

r_i - amplitudes



<u>Valid gate if:</u>

$$\sum_{i=1}^{N} \frac{r_i^2}{n_i} = 1 \qquad \xi_0 = 2\eta\Omega = \frac{2\pi}{T}$$

$A(T) = \pi/2$ $\qquad$ $F(T) = G(T) = 0$

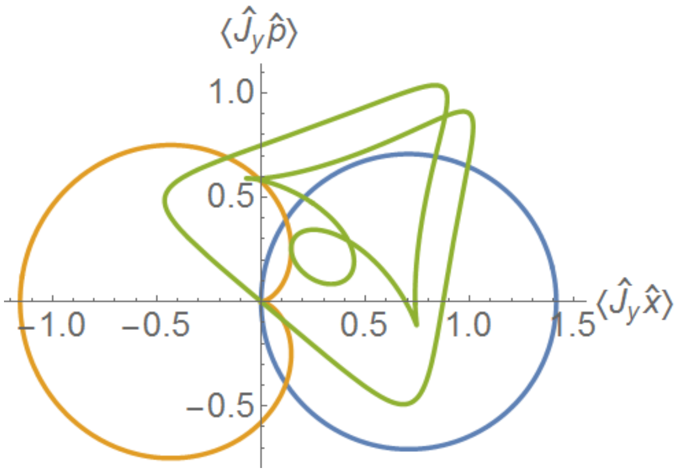
Multi-tone Mølmer – Sørensen gates

Continuous family of gates: $\{n_i\}_{i=1}^{N}$ $\{r_i\}_{i=1}^{N}$

$$\hat{U}(t;0) = e^{-iA(t)\hat{J}_y^2} e^{-iF(t)\hat{J}_y\hat{x}} e^{-iG(t)\hat{J}_y\hat{p}}$$

- Choose your favorite shape through phase-space
- Extra degrees of freedom:

  Overdetermined system; add constraints



Optimization of Quantum Gates

Analytic expression for gate fidelity:

$$F_g = \frac{3 + e^{-4\left(\bar{n}+\frac{1}{2}\right)\frac{F^2+G^2}{2}}}{8} + \frac{e^{-\left(\bar{n}+\frac{1}{2}\right)\frac{F^2+G^2}{2}} \sin\left(A + \frac{FG}{2}\right)}{2}$$

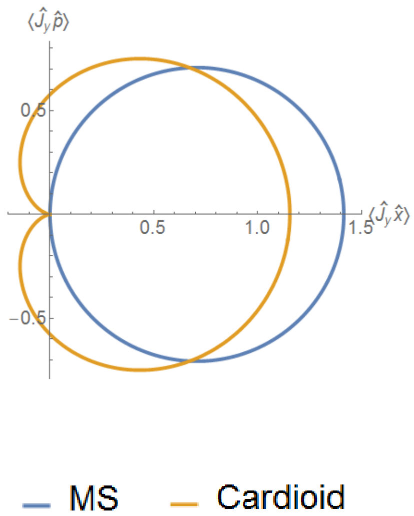
Null errors order-by-order:

$$F_g = 1 + \frac{1}{2}\frac{\partial^2 F_g\left(\{n_i, r_i, \phi_i\}_{i=1}^{N}\right)}{\partial \alpha^2}\bigg|_{\alpha_{ideal}} \delta\alpha^2 + \cdots$$

Gate timing errors – Cardioid gates

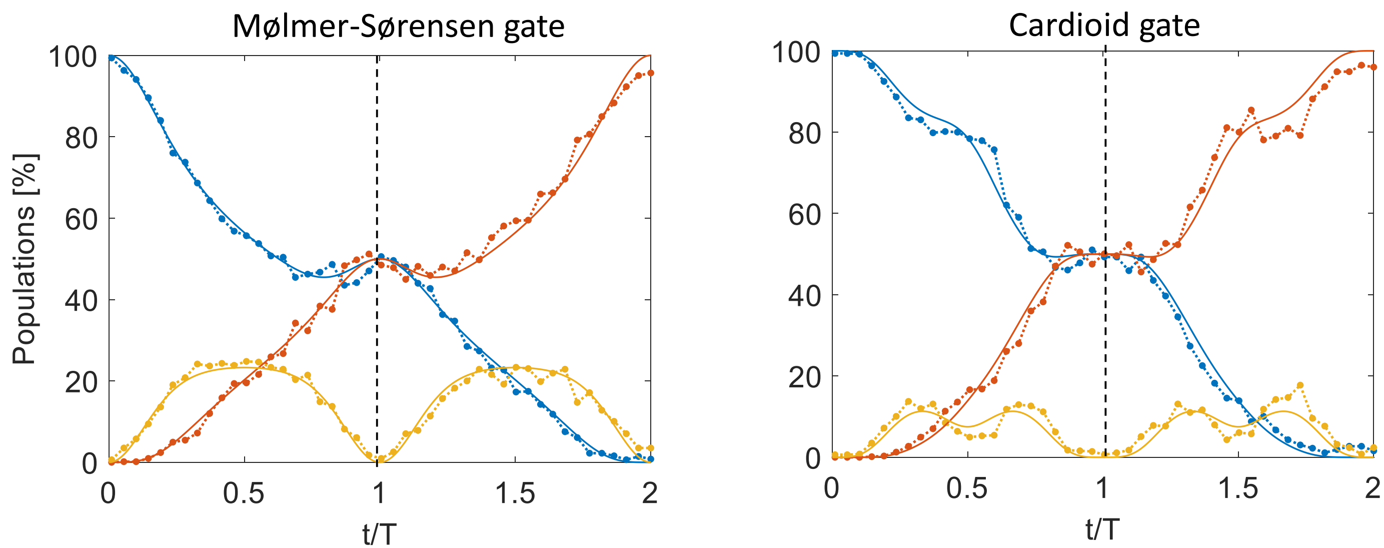
$$T = T_0 + \delta T \quad \Rightarrow \quad 1 - F_g \sim \left(\frac{\delta T}{T}\right)^{2N}$$

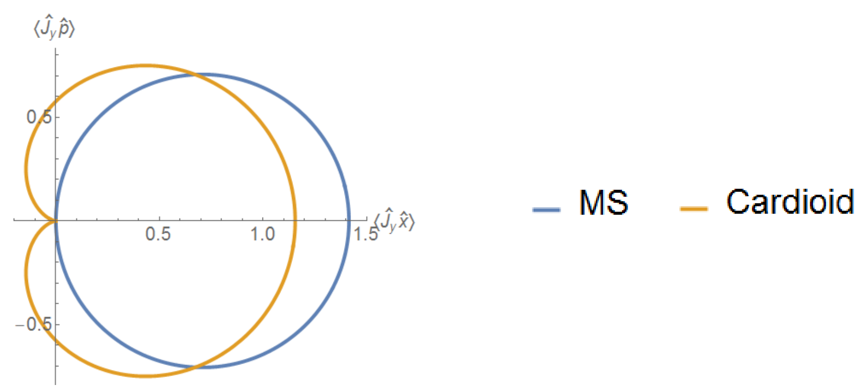
$N = 2$ Solution is a Cardioid $r_1 = -r_2$



Gate timing errors – Cardioid gates

Gate fidelity:

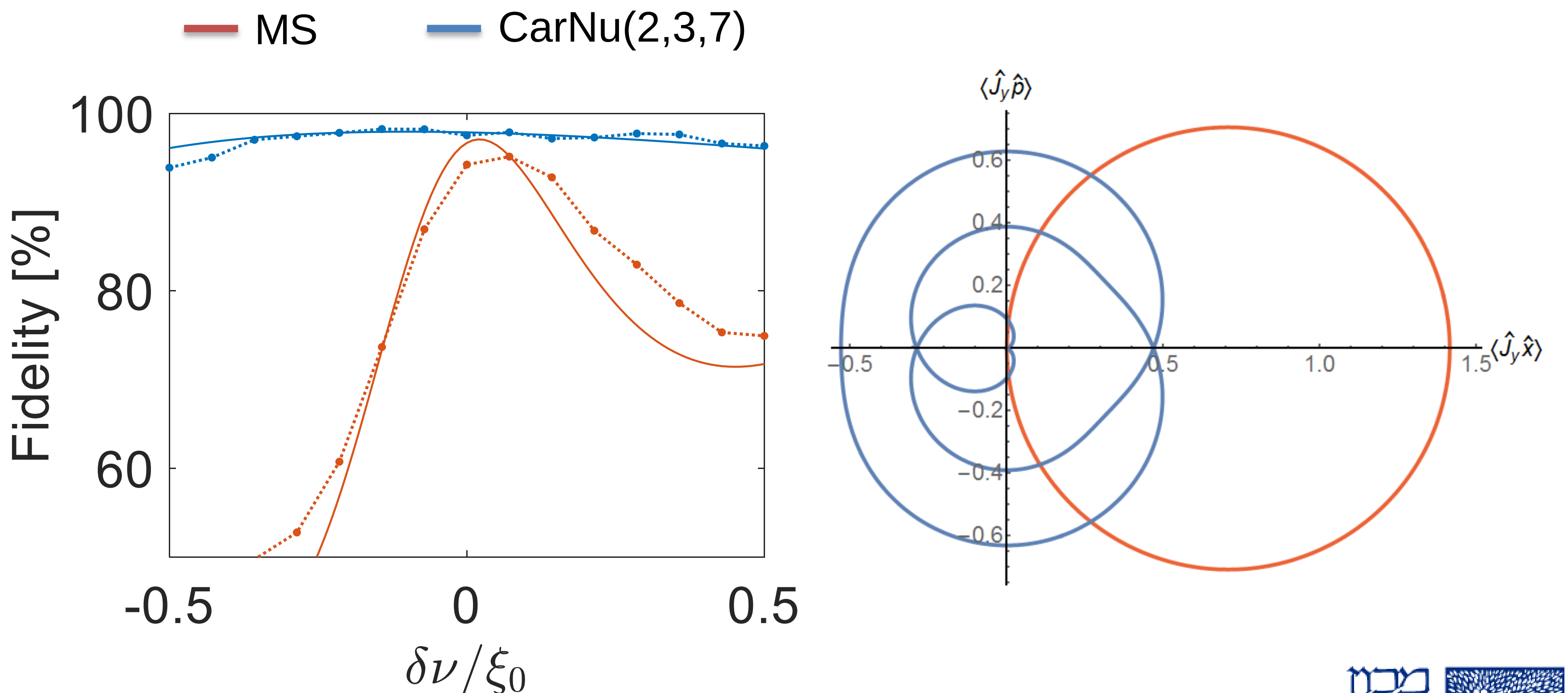




Shapira, Shaniv, Manovitz, Akerman, RO, Phys. Rev. Lett. 121, 180502 (2018)

Trap frequency errors – CarNu gates

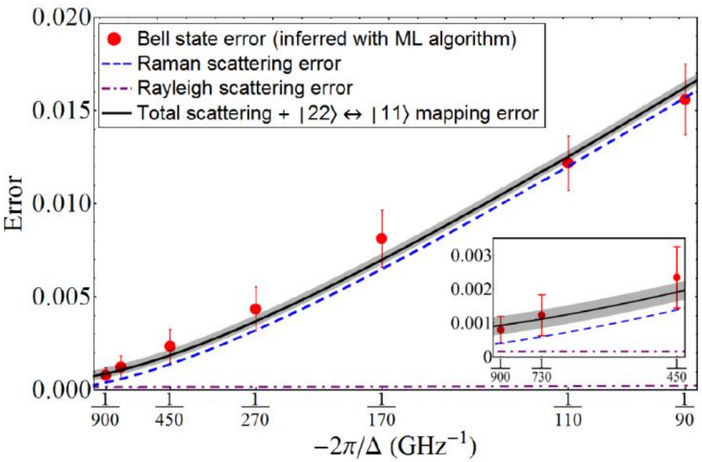
Trap frequency and timing errors: $\nu_0 + \delta\nu$ $T = T_0 + \delta T$



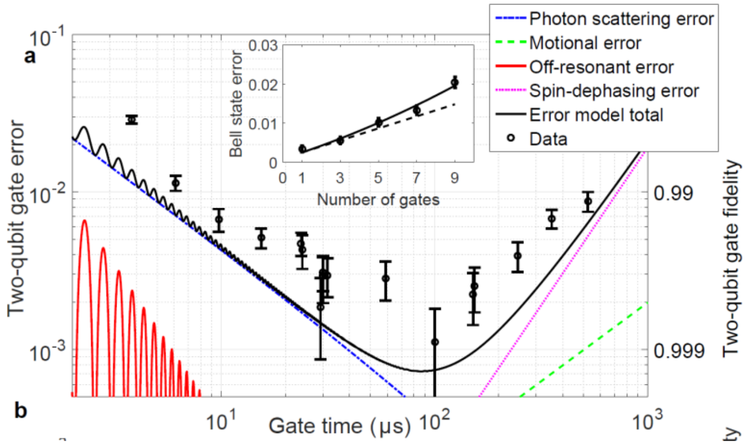
Shapira, Shaniv, Manovitz, Akerman, RO, Phys. Rev. Lett. 121, 180502 (2018)

Two-qubit gate fidelity

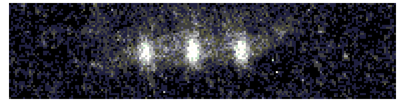
NIST

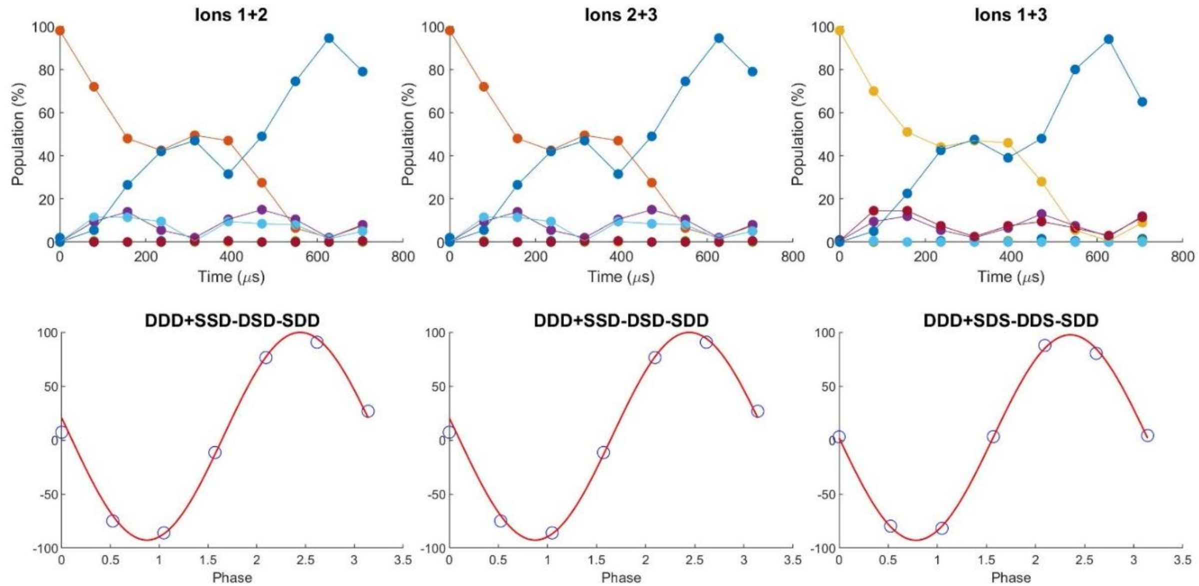


Oxford



Entanglement gates in multi-ion crystals





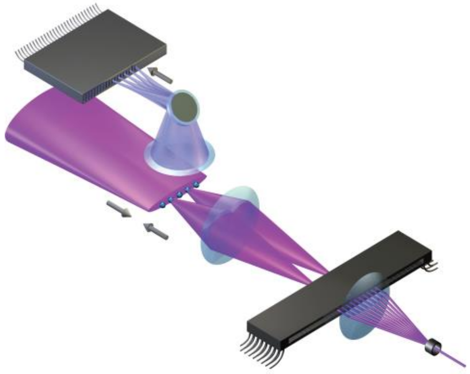
- Multiple modes of motion: choose one
- Slower because of heavier effective weight
- Increasing laser-power: off-resonance coupling to other modes or carrier transition
- Still works for a small number of ions

Small Scale Quantum computers

LETTER
doi:10.1038/nature18648

Demonstration of a small programmable quantum computer with atomic qubits

S. Debnath[1], N. M. Linke[1], C. Figgatt[1], K. A. Landsman[1], K. Wright[1] & C. Monroe[1,2,3]

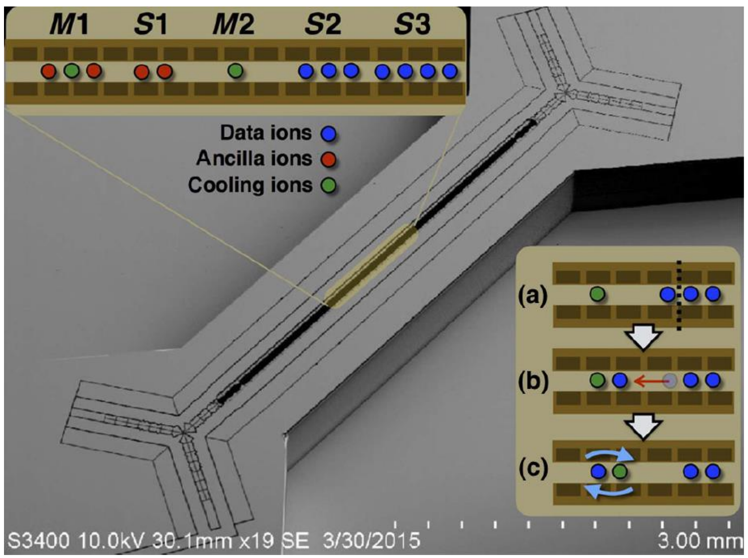


Quantum Algorithms demonstrated:

- Quantum Chemistry Calculations *Phys. Rev. X (2018)*
- Grover Search algorithm *Nature Comm. (2017)*
- Topological Quantum error-correction *Science (2014)*
- Shor Factoring Algorithm *Science (2016)*
- *Many more…*

Architectures for scaling up: # 1

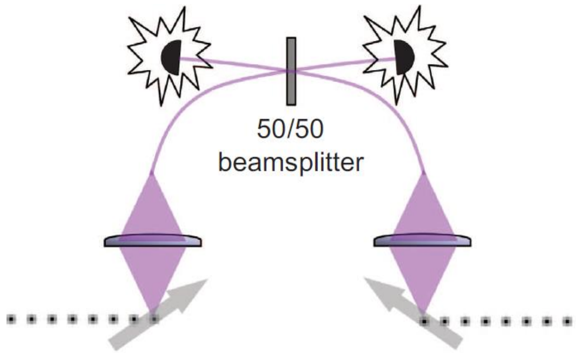
Shuttling ions between different trapping regions

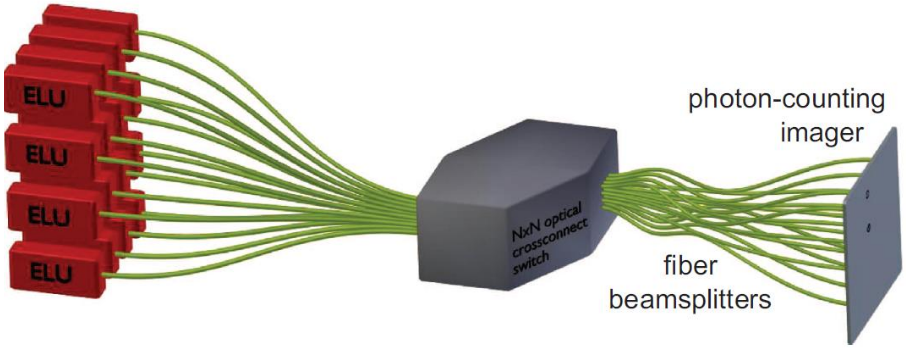


Kielpinski, Monroe and Wineland *Nature* 417, 709 (2002)

Architectures for scaling up: # 2

Photon interconnects between elementary Logic units (ELU's)

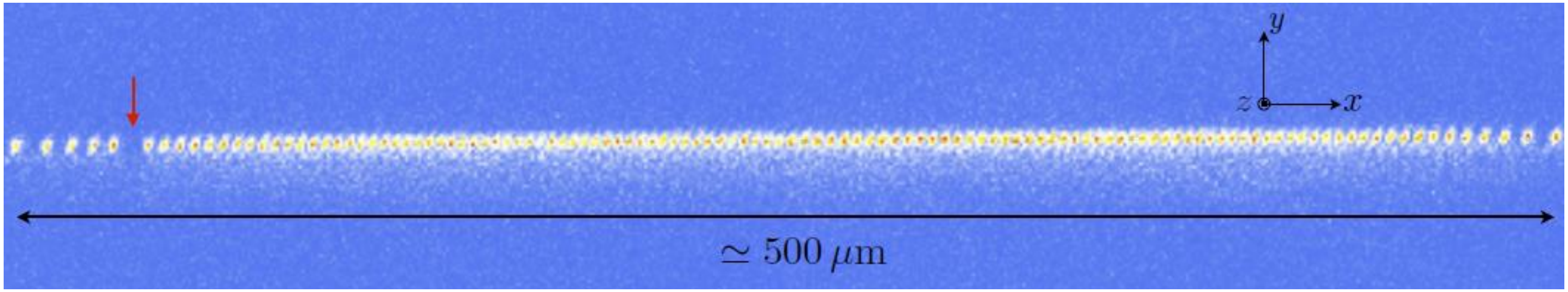




Hong-Ou-Mandel interference

Monroe *et. al. Phys. Rev. A* 89, 022317 (2014)

NISQ approach: Large Crystals



Monroe, JQI

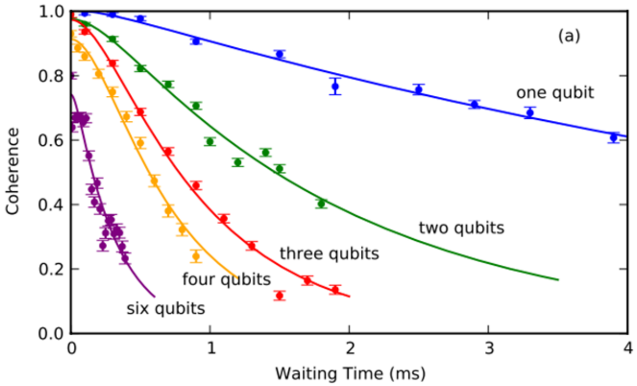
10 ’s -100’s ions

- Exponential speed-up for quantum computing ~ 60 ions sufficient, provided high fidelity
- Quantum simulators
- Can we stretch previous ideas to larger ion-numbers?

NISQ approach: Large Crystals

<u>Challenges:</u>

- 10’s – 100’s of motional modes; spectral crowding
- With 2-qubit gates many concatenated operations are required (N^3 in Shor’s)
- In the MS gate and its variants the gate time $T \propto 1/\sqrt{N}$
- Infidelity, $F_g \sim 1 - \frac{T}{T_2} N^2$
- Increasing laser-power: off-resonance coupling



- Possible remedy: parallel gates

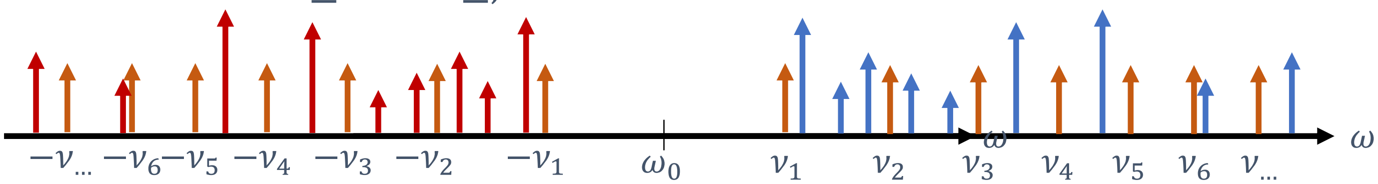
Use Multiple modes of motion

J. I. Cirac & P. Zoller, PRL 74, 4091 (1995). [3] T. Monz et al. PRL 106, 130506 (2011).

Quantum Engineering of multi-mode Gates

Multi-tone MS

- We generalize, $\omega_{\pm} \rightarrow \omega_{\pm,i} = \omega_0 \pm \omega_i$ at amplitude r_i and phase ϕ_i.



- The Hamiltonian generalizes:

$$\hat{H} = \hbar \sum_{j=1}^{N} \hat{J}_{y,j}\left(f_j(t)\hat{x}_j + g_j(t)\hat{p}_j\right), \quad \hat{J}_{y,j} \equiv \frac{\sqrt{N}}{2}\boldsymbol{v}_j \cdot \boldsymbol{\sigma}_y$$

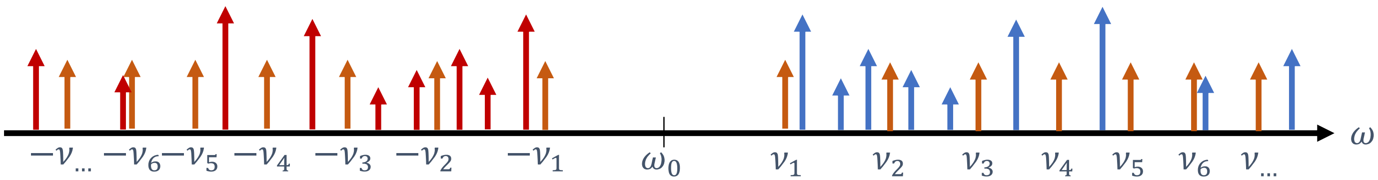
Normal-mode vector

- With: $f_j(t) + ig_j(t) = \frac{2\sqrt{2}}{\sqrt{N}}\eta_j \Omega \sum_{i=1}^{M} r_i \cos(\omega_i t + \phi_i) e^{i\nu_j t}$

Normal-mode frequency

Quantum Engineering of multi-mode Gates

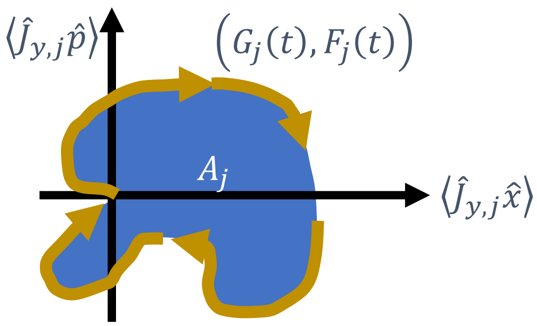
Multi-tone MS



- Similar unitary:

$$\hat{U}(t;0) = \prod_{j=1}^{N} e^{-iA_j(t)\hat{J}_{y,j}^2} e^{-iF_j(t)\hat{J}_{y,j}\hat{x}_j} e^{-iG_j(t)\hat{J}_{y,j}\hat{p}_j}$$

- Same phase-space intuition



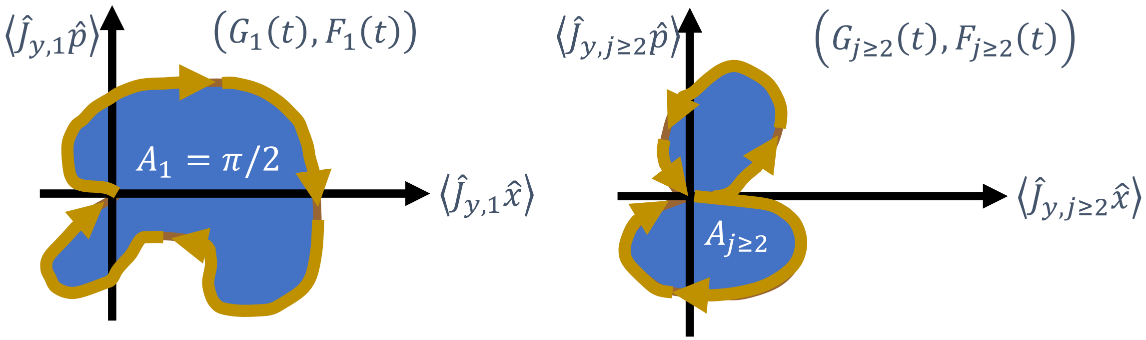
Constraints – phase space closure

•Motion closes in all phase−spaces at the gate time. <u>Linear constraint</u>

$$F_j(T) = G_j(T) = 0 \text{ for all } j = 1, \dots, N.$$

•Geometric phases for all-to-all coupling: <u>Quadratic constraint</u>

$$\mathrm{A}_1(\mathrm{T}) - A_{j\geq 2}(T) = \frac{\pi}{2}$$

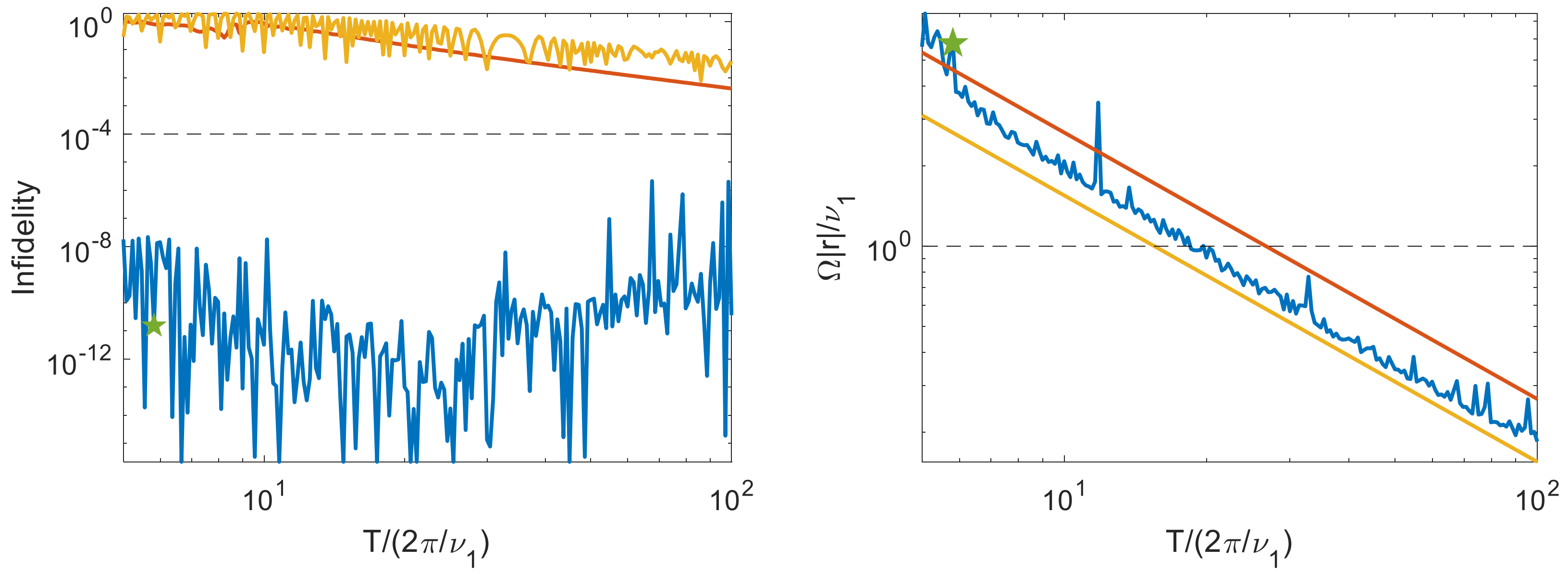


The quadratic constraint optimization problem is NP-Hard

Quantum Engineering of multi-mode Gates

Gate optimization results – 6 ions, axial modes

Blue – Multi-mode gate, Yellow – regular MS, Red – multi-tone adiabatic

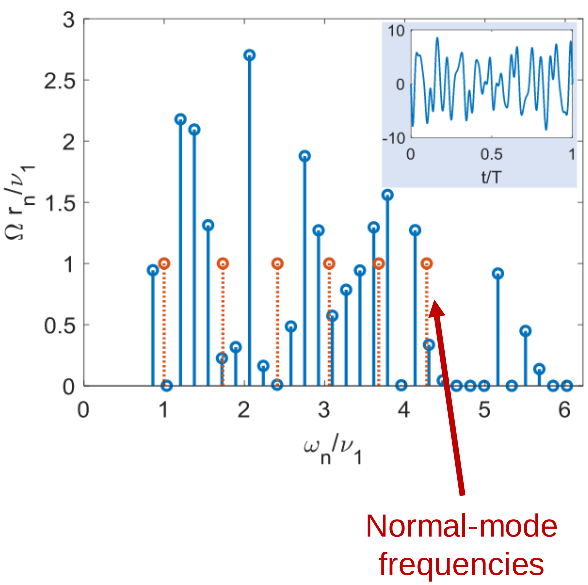


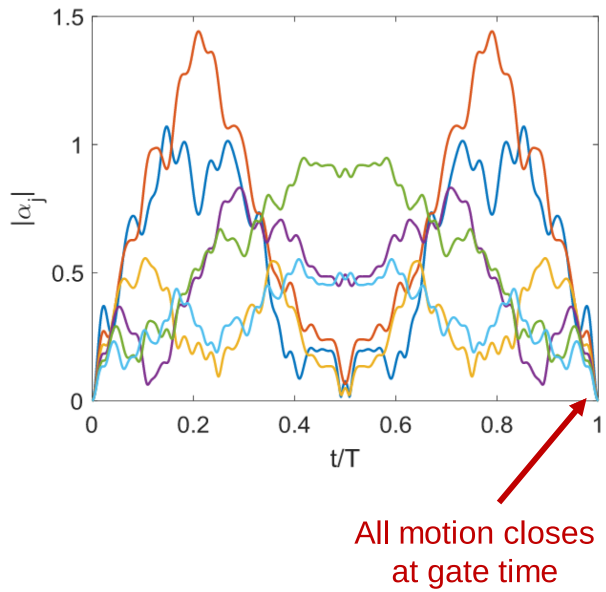
Quantum Engineering of multi-mode Gates

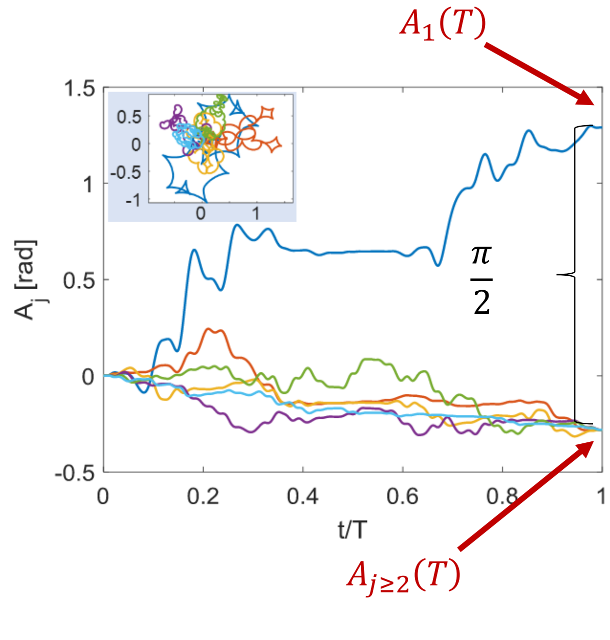
Gate simulation results – 6 ions, axial modes

22 tone pairs, robust to: timing errors, normal-mode frequency drifts, heating

$$T \approx 5.8\,\frac{2\pi}{\nu_1},$$



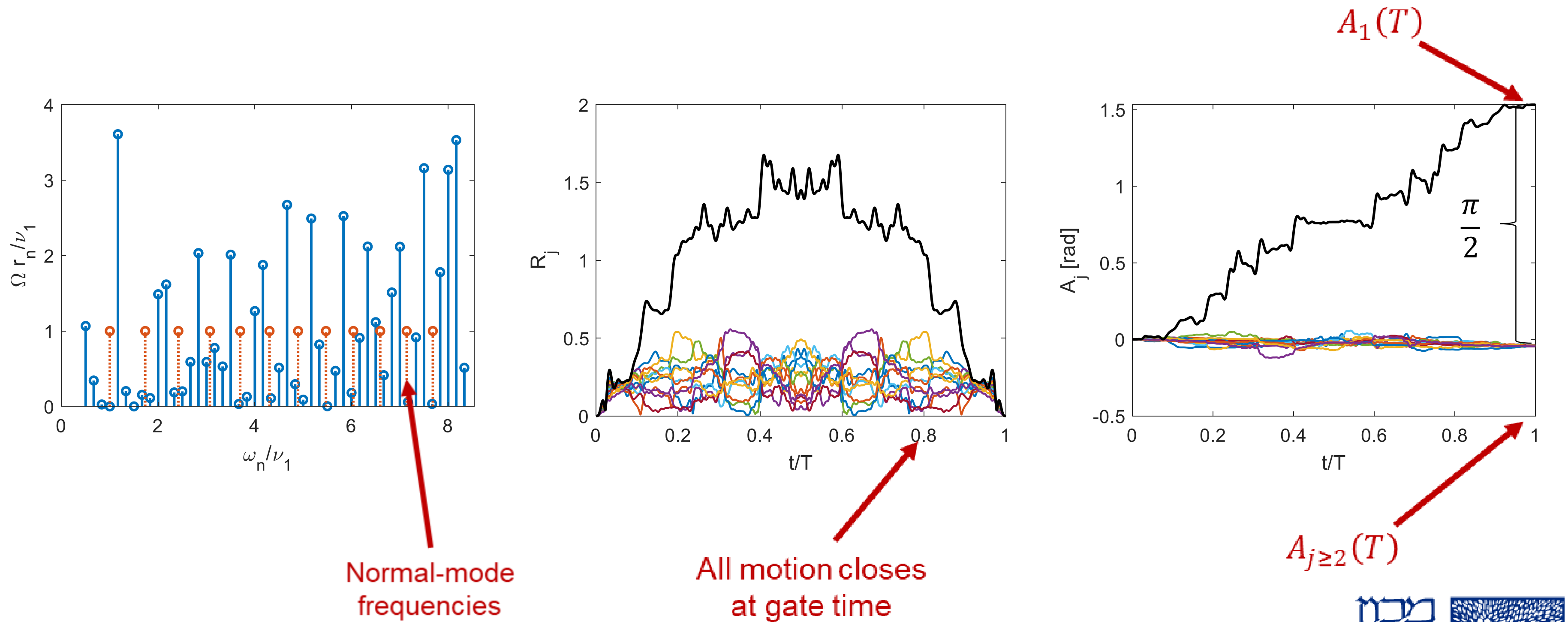




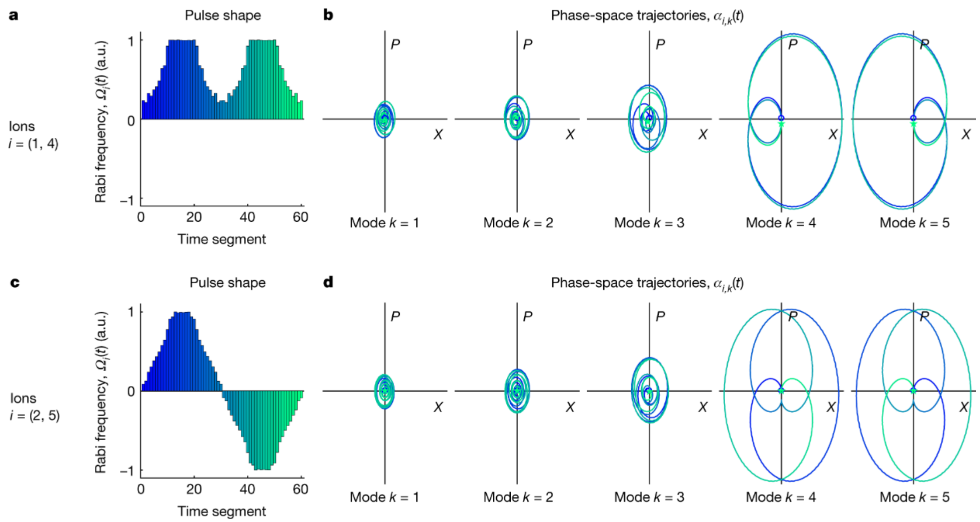
Quantum Engineering of multi-mode Gates

Gate simulation results – 12 ions, axial modes

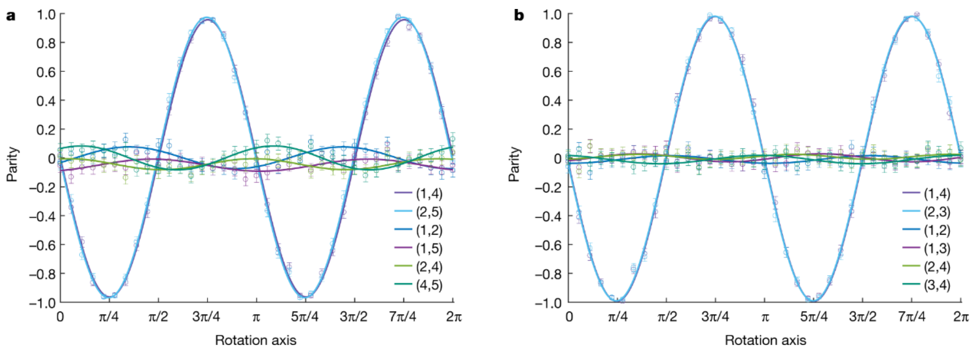
- $T \approx 6\frac{2\pi}{\nu_1}$, 41 tone pairs, robust to: timing errors, normal-mode frequency drifts, heating. Resulting fidelity is 0.9987.



Experimental implementation of similar ideas

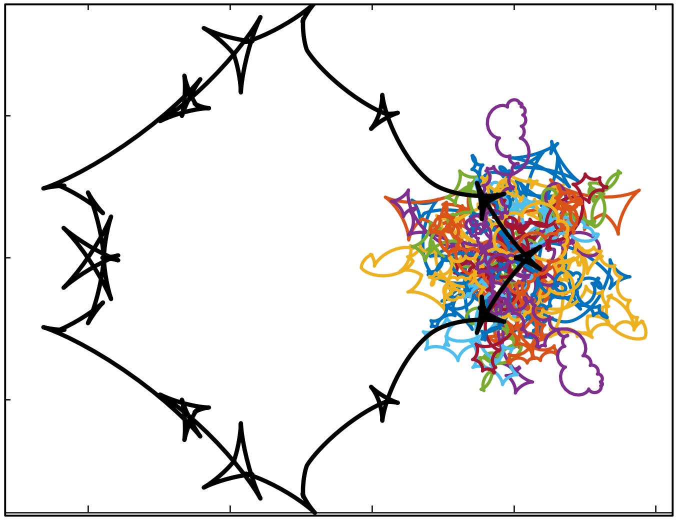


Monroe
JQI and IonQ



C. Figgatt et al. Nature 572, 368 (2019). Y. Lu et al. Nature 572, 363 (2019)

In Summary



- Trapped-ion quantum computers:

  High-fidelity quantum gates

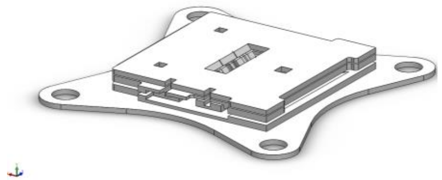
  Small universal quantum machines

- Increasing to larger ion-qubit numbers:

  Scale-up architectures: interconnected small modules

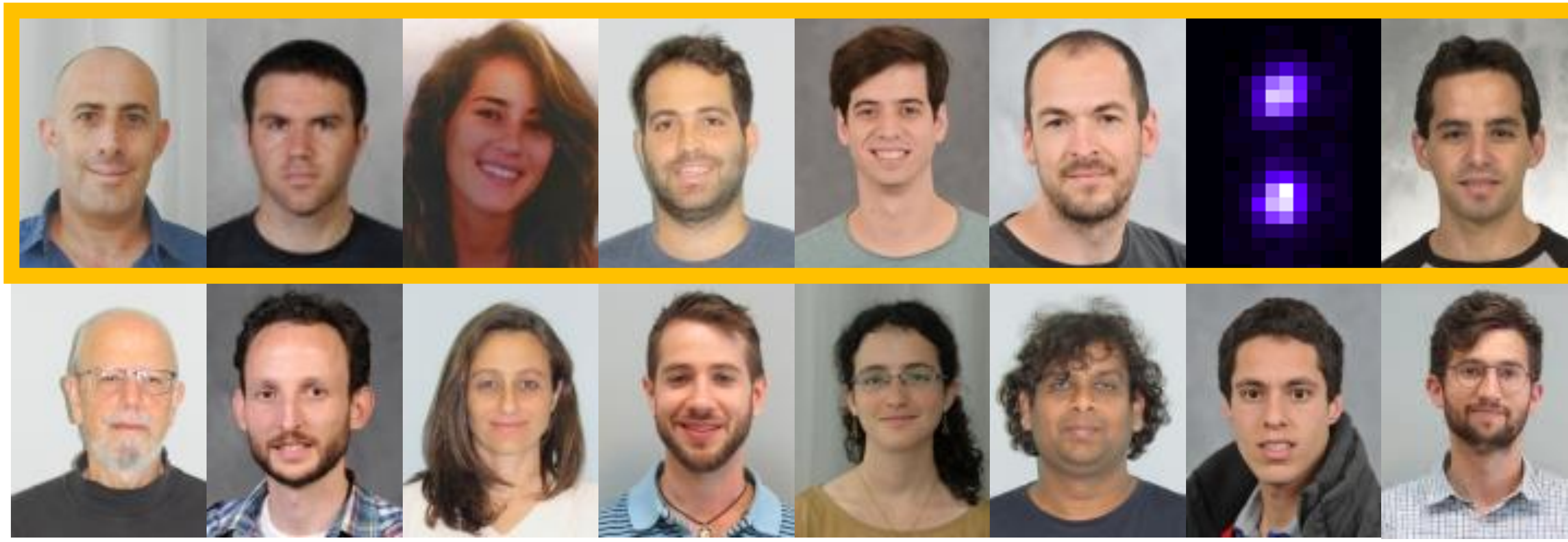
  Large-ion crystals: Coherent control optimization problem - hard, yet possible

- Work towards ~100 qubit trapped-ion universal quantum computers



The Weizmann trapped ion team

Roee Ozeri | Ravid Shaniv | Lior Gazit | Lee Peleg | Tom Manovitz | Yotam Shapira | Sr^+ ions | Nitzan Akerman



Avi Gross | Meir Alon | Ruti Ben-Shlomi | Haim Nakav | Meirav Pinkas | Vidyut Kaushal | Or Katz | David Schwerdt

Ady Stern

AMOS Atomic Molecular Optical Sciences

ISRAEL SCIENCE FOUNDATION

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