QSS08 - Thierry Giamarchi - Questions & Answers

Thierry Giamarchi

THIERRY: First, many thanks to all for these very nice (and challenging !) questions both during the talk and below. Hope that the few lines below answer your questions. In case this is not the case don't hesitate to send me an email.

William Phillips: There are various senses in which systems can be 1-D or point contacts. Could you say a bit about this for atoms and for CM?

THIERRY: The first condition is of course that the quantization due to the confinement in the directions transverse to the "tube" leads to energy gaps that are much higher than the temperature and/or interactions. In that case one can safely ignore the transverse directions and focus on the longitudinal degrees of freedom. Then the condition to decide between 1D and 0D (QPC) is essentially similar depending on the length of the system. There is a characteristic length on which the properties of the system can vary. For a superconductor for example this is the coherence length (would be the healing length for bosons). Then if the system is long compared to this coherence length the variation of the properties along the tube must be considered and the system is a 1D system. On the contrary if the length is short, then one can consider that the system is zero dimensional. For condensed matter the coherence length is usually quite long since Tc is much smaller than EF, the Fermi energy – typically 10 K versus 10 000K, thus ξ_0 the coherence length can be large (e.g. 10000 Å) . This allows to get QPC in break junctions such as the ones shown on slide 7. Conversely realizing good long 1D superconducting wires is not so easy for the same reason, since one need to realize a "long" wire on which the transverse confinement is regular. As a result many superconducting "wires" have turned out to be in the end in the regime of temperature at which they were considered more describable in terms of 0D structures. Doing good 1D superconducting wires is thus a very challenging and interesting question. For cold atoms, the attraction can be made very strong, which means that one can have relatively small cooper pairs if one wants to. Of course the possibility to have a good point like attraction is unique. The transverse confinement can be realized in an efficient and controlled way. So doing either a QPC or a 1D structure is in principle possible. The price is, like with most fermion experiments, the fact that the temperature itself is still quite high (still of the order of TF/10). The cold atomic system have thus provided very nice and complementary systems to the CM ones and allowed to probe both the QPC and 1D regimes, but with the obligation for the theory to be serious about the temperature effects.

Karen Hallberg: Hi Thierry, thanx for your interesting talk! What about charge transport with repulsive interactions?

THIERRY: The same (theoretical) technology can be used for computing transport with repulsive interactions. In that case for the charge transport properties are of course quite sensitive to the filling of the band, leading to a Mott insulator at half filling. The interesting aspect is that one can use the duality existing with contact interactions (exact for the Hubbard model, and approximately exact for a Tomonaga-Luttinger liquid) allowing to map the repulsive interactions to attractive ones. This is obtained by doing a particle-hole transformation on one spin species only. In that case the charge sector becomes the spin sector. The spin transport with attraction is thus directly related to the charge transport with repulsion. The backscattering term with exists for the spin transport for the unpolarized system is like the Umklapp term that exists in the charge transport for 1/2 filling and that is responsible for the Mott insulating phase.

Ekaterina Plotnikova: I wonder what would happen if the weak periodic potential is tilted? William Phillips: Following up on the question of Ekaterina Plotnikova about a tilted potential—when would you get Bloch oscillations instead of conduction?

THIERRY: Things will depend on what is inside the system (e.g. clean or disordered etc.). For a clean, non-interacting system the tilt of the potential will lead ultimately to Bloch oscillations, since the particles will "accelerate". Note that if the system is attached to reservoirs then the potential along the system itself remains zero since the system is a perfect conductor (R=0 thus V= R I means zero potential even with a finite current). All the potential drop takes place at the contact between the reservoirs and the system. Thus applying the Laudauer-Buttiker formula leads to a perfect conductance and a finite current. This is different from the situation with the Bloch oscillations. If the system contains processes that lead to scattering, either due to interactions or e.g. to disorder, then one would expect that system to have a finite resistivity and thus to have a uniform potential drop inside from +V/2 to -V/2. In that case the tilting of the potential and the reservoirs should be more closely related, but of course because there are scattering the Bloch oscillations are also destroyed. There can also be, depending on how the system can locally equilibrate inside the wire, non equilibrium distributions (e.g. a double Fermi step) if inelastic scattering do not exist. These out of equilibrium distributions have been measured in condensed matter wires. Globally how perfect systems (e.g. interacting but otherwise clean and invariant by translation – integrable or not) would react to a tilt and/or reservoirs is a non trivial and interesting question indeed !

W. Vincent Liu: Many-body insulator pinned by LE liquid is interesting. Can this be thought as due to the size of Cooper pair in ID (uncondensed!) being shorter than that of weak BCS superconductor in 3D? I would like to ask your expert opinion on what might be a good scale to think of the size of the 1D Cooper pair. Seems very different than 3D weak BCS case (with coherence length scale).

THIERRY: No the pinning of the LE liquid is not directly connected to the issue of the size of the cooper pair. It comes from the fact that in 1D because susceptibilities are in general divergent even a very small perturbation can lead to an ordering. For example for bosons if one puts a very small periodic potential with 1 boson per period one can get a Mott insulator of bosons, provided the repulsion between the particles is large enough. This would be impossible in 2D/3D and even if you have hard core bosons you would need a deep enough lattice to be able to go from the superfluid to the Mott insulator. The pinning of the LE rests on a similar property. Because there is spin charge decoupling in 1D you can think of the cooper pair (regardless of its size) as a kind of effective hard core boson (made of two original fermions). These bosons are "hard core" and have in addition a longer range repulsion (one pair near another pair would block virtual breaking and recombination of the pair – hence a loss of energy of the order of $t^2/|U|$). Thus the filling of two fermions per period of the potential correspond to 1 "boson" / period and will lead to a 1D Mott insulator of bosons.

For the size of the cooper pair itself, one can have a priori any size depending on the interaction U. Comparing between 1D and 3D would depend on the precise density of states considered but actually since in 1D the susceptibilities are logarithmically divergent, the equation for the gap in spin sector is quite similar to the mean field BCS equation in 3D (with the different density of states).

François Damanet: Regarding the spin transport work: in order to reach the interesting regime experimentally, would it not be easier to reduce the size of the channel instead of reducing the temperature?

THIERRY: Indeed, the difficulty is to find the sweet spot in which you make the channel long enough that you have more than a couple of particles in it but short enough that the thermal length allows to scan the different regimes. One can also play with the cooper pair size.

Ramesh Thamankar: from the point of view of experiment, the channel considered is very long. would you not get any defect in this channel ?

THIERRY: The correct answer should come directly from the experimentalists on that point :-) but I would say that the control on the cold atom system is good enough that there are no defects in the sense of a strong bump of the potential etc. This problem is conversely absolutely crucial for the condensed matter realizations for which if you want a good 1D wire you must have a quite tight transverse confinement but at the same time ensuring that you are not pinching the wire at a point. This has happened many times in the past and is one of the obstacles in realizing good 1D superconducting wires. This is even worse if you don't want the wire to be superconducting since in that case you must compare the transverse dimensions to k_F^{-1} and not to the coherence length which puts much more severe constraints .This could however be done in semiconducting systems which offer a good enough control on the structures.

Jinlong Yu: Thanks Thierry. I wonder if the 1D contact is topological itself (e.g., forming an SPT such as an SSH lattice), does the transport change a lot?

THIERRY: That is a very good question and I don't know the answer but would be very tempted to answer yes. Indeed we know that if we get Andreev bound states there are consequences. Certainly something to explore.

Gal Ness: Thanks for an inspiring talk! Can you remark about the case where one periodically flips the direction of the driving mu difference, studying the ”AC” voltage characteristics?

THIERRY: A priori the answer is yes, and this would be a measure of the a.c. conductivity. In the experimental system I was presenting in the talk, this would be difficult to do in practice, since the system is originally prepared with an imbalance of population and then simply released to the channel. For the a.c. conductivity however there are other ways to realize such measurements. One simple way (see e.g. A. Tokuno + TG PRL 106, 205301 (2011)) is to put the system in an optical lattice and then modulate the phase of the optical lattice (or conversely shift periodically the center of the trap – which corresponds to a change of referential see Z. Wu, E. Taylor, and E. Zaremba, EPL 110, 26002 (2015)). This produces the equivalent of a periodic force on the system and gives access (in principle :-)) to the a.c. conductivity. In practice this has been used in cold atoms (see e.g.g R. Anderson, F. Wang, P. Xu, Vi. Venu, S. Trotzky, F. Chevy, and J.H. Thywissen PRL 122, 153602 (2019)).

On the theory side computing the a.c. conductivity is of course an interesting question. Quite paradoxically this is in fact less difficult that computing transport at $\omega=0$ since the frequency provides a good cutoff on the time dependence. In an interacting 1D system on typically gets (this depend of course on the system) powerlaws of the frequency with exponents that depend on the interactions inside the system (the so called Luttinger liquid exponents). For more details on that point I can refer you to the transport chapter in my book on 1D systems.

Jeremy Levy: Hi Thierry, thank you for the talk! I was wondering if it makes sense (is possible) to think of spin drag in 1D as a type of Hall effect, using the trick of synthetic dimensions that you discussed.

THIERRY: Yes one can indeed create Hall effect measurements by measuring the spin transport. The spin up and down would be the upper and lower leg of a two leg ladder and measuring the charge/spin transport gives access to the current on each "leg" separately. In order to make it a ladder and to have a Hall effect one need to make transitions that flips the spin with a phase (as e.g. in Fallani's experiment on slide 28). Then the spin-drag is indeed practically the Hall resistance (if one normalized by the charge and spin conductance). The only catch is that there are "non-local" (i.e. coupling the two "legs" of the ladder) interactions that come from the fact that in the true system spin up and down share the same physical space, but this is not necessarily a problem (more a feature than a bug :-))

William Phillips: This talk has been about fermions. What about bosons? We don't have a fermi energy or pairing, but we do have superfluidity and features like transverse excitation quantization in a channel, so what kinds of things are the same and what are different?

THIERRY: Actually in 1D fermions and bosons are not that different :-) There is a the equivalent of a "Fermi wavector" with the identification of $2\ k_F = 2\ \pi\ \rho_0$ (ρ_0 being the average boson density). This is apparent when one uses the bosonisation technique to describe such systems and also of course via Jordan-Wigner transformations (the Tonks limit is one of the examples of this connection). Loosely speaking fermions with attraction are very close to bosons with repulsion in 1D. One has thus very related phenomena for transport in interacting bosonic systems and several aspects of the transport are indeed very similar (e.g. anomalous powerlaws, pinning on weak lattices etc). For bosons for example a good geometric to tackle such issues could be ring geometries. The differences between fermions and bosons will become much stronger when one has to deal with coupled 1D systems, because in that case the mapping does not hold and coupled 1D chains of fermions are a priori quite different (in particular with repulsive interactions) than coupled chains of bosons. This means that transport through ladders, quasi-1D systems and Hall are of course interesting to understand and will a priori depend much more on the statistics of the particles.

William Phillips: You use the terms superconductor and perfect conductor, and these are different. Are they also different in cold atoms as opposed to condensed matter fermion systems?

THIERRY: Yes they are. Free fermions are a perfect conductor, but the system would not have any phase rigidity contrarily to a superconductor. So although both would have at T=0 formally an infinite conductivity several other properties would be markedly different such as e.g. the response to a twist in boundary condition or equivalently the current induced by a flux in a ring geometry. The periodicity with flux would be different for the two systems. They are also different in their finite temperature behaviors and also if one puts a small amount of scattering in the system. The superconductor is of course usually (1D is a more complicated case here) more robust than a perfect conductor. For the last part of the question: I don't think there would be any difference of principle between cold atoms and condensed matter, with perhaps the two following "limitation" that in condensed matter it is practically impossible to realize a perfect conductor, since there is always in practice something to scatter the carriers. The best that has been achieved in that respect is semiconducting structures that are clean enough that the mean-free path is larger than the size of the system but this is more the exception than the rule. Cold atoms provide much more easy paths to realize "perfect conductors".

Yaakov Yudkin: This setup looks like a macroscopic double-well potential. Are the phenomenology and theory similar? And could you e.g. see Rabi flopping in Esslinger's experiment?

THIERRY: It is a double well potential only if you consider that only the global phase of the reservoirs matters. In that case you would get indeed Josephson oscillations due to the potential difference between the two wells. These oscillations have been seen in a slightly different setup where the junction between the reservoirs is long and the tunneling quite weak so that mostly the Josephson coupling between the reservoirs is present and the single particle tunneling is largely suppressed [see G. Valtolina, A. Burchianti, A. Amico, E. Neri, K. Xhani, J. A. Seman, A. Trombettoni, A. Smerzi, M. Zaccanti, M. Inguscio, G. Roati Science 350 1505 (2015)]. In the case of the experiment that I was mentioning by the Brantut/Esslinger group the tunneling is quite good and thus the single particle excitations should be taken into account. One should thus also consider these excitations in the reservoirs. This leads to a very interesting mechanism in which the transfer of a pair from one reservoir to the other can instead of leading to oscillations, give its energy to a single particle excitation (and this repeated for many pairs) to overcome the existence of the single-particle gap. This mechanism (multiple Andreev reflections) thus allows to have a d.c. current at finite voltage (even if this voltage is below the gap).

Emily: (1) can you speak about difference of various results if interactions are long-ranged? (2) if I want to play around with DMRG, is there a publicly available code you recommend?

THIERRY: The long range effects is of course an interesting question whose answer would need much more than those few lines. In short modifications with depend on how "long-range" is the long-range interaction. Typically the long range interaction (and I assume here that you mean long range repulsive) would lead similar effects that more "repulsive" contact interactions provided the interaction is integrable, namely that $\int dr\, V(r)$ is finite. This would be the case for example for a dipolar interaction (which decreases as $1/r^3$) in 1D. These effects that be incorporated in a modification of the Luttinger liquid parameters. Of course depending on how strong is this modification other phases (that would be inaccessible with just a contact interaction) can be explored and thus the transport is modified accordingly. The situation becomes more extreme when the interaction is not integrable (which is the case of the Coulomb 1/r interaction). In that case one has a modified phase (which was called a Wigner crystal), which has different properties than most of what I discussed in the talk, with a much more ordered charge structure (this more easily pinned on potentials either periodic or disordred). I refer you to a chapter ("Refinements") in my book on 1D systems if you want to know more on this particular case.

For the DMRG I guess that there are several possibilities. One quite complete numerical library is the ALPS library http://alps.comp-phys.org/mediawiki/index.php/PapersTalks which has several codes for numerically solving models (exact diagonalization, monte-carlo, and of course a DMRG code). Another extremely useful library is the Itensor library which contains both the tools and tutorials on DMRG http://itensor.org/

Callum Duncan: Thinking along the lines of the scanning tunnelling microscope, could you use this kind of set up as a probe of some properties of what is at the moment one of the reservoirs? Or is this not a useful line of thought in a cold atom set up? I would perhaps assume the latter.

THIERRY: At the moment the present setup is more used to probe the physics of the system between the two reservoirs. But you are perfectly correct that this is also a setup in which one does probe the physics of the reservoirs (for example the observation of the multiple Andreev reflections with the QPC is clearly more linked to the properties of the reservoirs than the "system" since the system is just vacuum :-) in that QPC case. One can thus in principle imagine using such systems in the same spirit than an STM. This could complement other techniques that have been used (such as e.g. the so called quantum microscopes, in which one has access to the density at each point of the system) to probe the bulk exotic systems.