Quantum computing over the rainbow: 
the quantum optical frequency comb as a platform for 
measurement-based universal quantum computing

Olivier Pfister
University of Virginia

Quantum Fields and Quantum Information website:
https://sites.google.com/view/qfqi
Quantum computing over the rainbow: the quantum optical frequency comb as a platform for measurement-based universal quantum computing

OR

How I learned to stop worrying and love the comb

Olivier Pfister
University of Virginia
The QFQI group

Rajveer Nehra
Xuan Zhu
Chun-Hung Chang

Rafael Alexander (now UNM)

Aye Win (now U.OK.)

Carlos González Arciniegas

Miller Eaton
Current projects

- Large-scale cluster-state entanglement
- Quantum state engineering and characterization (quantum error correction encodings)
- Quantum photonics on chip. Collab. w/ UVA ECE: Campbell, Beling, Yi
- Quantum simulation of nuclear physics
- Quantum simulation of condensed matter physics

Today I will address the first two topics.
Building a continuous-variable quantum computer with light

Gaussian quantum optics (fields)
• Large-scale entanglement (squeezing)
• No postselection

Non-Gaussian quantum optics (photons)
• Exponential speedup
• Quantum error correction
Two main flavors of universal quantum computing

1. Circuit-based

![Circuit-based quantum computation diagram]

2. Measurement-based

![Measurement-based quantum computation diagram]
Two main flavors of universal quantum computing

1. Circuit-based

2. Measurement-based
Two main flavors of universal quantum computing

1. Circuit-based

2. Measurement-based
Two main flavors of universal quantum computing

1. Circuit-based

2. Measurement-based
Teleportation as a qubit gate primitive
Teleportation as a qubit gate primitive

\[ |\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle \]
Teleportation as a qubit gate primitive

\[ |\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle \]
Teleportation as a qubit gate primitive

\[ |\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle \]
Teleportation as a qubit gate primitive

$$|\psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle$$
Teleportation as a qubit gate primitive

\[ |\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle \]
Teleportation as a qubit gate primitive

\[ |\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle \]
Teleportation as a qubit gate primitive

\[ |\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle \]
Can also shape cluster states with measurements

\[|+\rangle \quad |+\rangle \quad |+\rangle \]

CZ  CZ
Can also shape cluster states with measurements

\[ |+\rangle |+\rangle |+\rangle \]

CZ  CZ

Measure Z
Can also shape cluster states with measurements

\[ |+\rangle |+\rangle |+\rangle \]

\[
\begin{array}{cc}
\text{CZ} & \text{CZ}
\end{array}
\]

feed forward

Measure Z
Can also shape cluster states with measurements
Can also shape cluster states with measurements

\[\ket{+} \ket{+} \ket{+} \]

CZ  CZ

feed forward

Measure Z
Can also shape cluster states with measurements

\[ |+\rangle |+\rangle |+\rangle \]

\[ \text{CZ} \quad \text{CZ} \]

feed forward

\[ \text{Measure Z} \]

\[ \text{Measure X} \]
Can also shape cluster states with measurements

\[
|+\rangle_1 \quad |+\rangle_2 \quad |+\rangle_3
\]

\[
\begin{array}{c}
\text{CZ} \\
\text{CZ}
\end{array}
\]

feed forward

Measure Z

feed forward

Measure X
Can also shape cluster states with measurements

\[ |+\rangle \quad |+\rangle \quad |+\rangle \]

\[ \text{CZ} \quad \text{CZ} \]

\[ \text{feed forward} \]

\[ \text{Measure Z} \]

\[ \text{feed forward} \]

\[ \text{Measure X} \]
Can also shape cluster states with measurements

$\vert+\rangle \quad \vert+\rangle \quad \vert+\rangle$

**CZ**  **CZ**

**feed forward**

**Measure Z**

**feed forward**

**Measure X**
Can also shape cluster states with measurements

$|+\rangle$  $|+\rangle$  $|+\rangle$

\[\begin{array}{c}
\text{CZ} \\
\text{CZ}
\end{array}\]

$\text{feed forward}$

$\text{Measure Z}$

$\text{feed forward}$

$\text{Measure X}$
Can also shape cluster states with measurements

$|+\rangle |+\rangle |+\rangle$

$\text{CZ} \quad \text{CZ}$

feed forward

Measure $Z$

feed forward

Measure $X$
Can also shape cluster states with measurements

\[ |+\rangle |+\rangle |+\rangle \]

CZ     CZ

feed forward

Measure Z

feed forward

Measure X

CZ
Discrete vs. continuous variables, qubits vs. qumodes

**Qubits**

\[ \{ |0\rangle, |1\rangle \} \]

e.g. eigenstates of Sz

\[ |\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle \]

Hadamard transform

\[ \{ |\pm\rangle \} = \{ |0\rangle \pm |1\rangle \} \]

e.g. eigenstates of Sx

\[ |00\rangle + |11\rangle \]

Bell state

**Qumodes**

\[ \{ |q\rangle \} \quad q \in \mathbb{R} \]

e.g. eigenstates of Q: position / amplitude quadr.

\[ |\psi\rangle = \int \psi(q) |q\rangle dq \]

Fourier transform

\[ \{ |p\rangle \} \quad p \in \mathbb{R} \]

e.g. eigenstates of P: momentum / phase quadr.

\[ \int |q\rangle_1 |q\rangle_2 dq \]

EPR state

\[ Q = \frac{1}{\sqrt{2}} (a + a^\dagger) \]

\[ P = \frac{i}{\sqrt{2}} (a^\dagger - a) \]

\[ [Q, P] = i \]

\[ \Delta Q \Delta P = \frac{1}{2} \]

**Discrete vs. continuous variables,**

**Qubits**

\[\{ |0\rangle, |1\rangle \}\]

e.g. eigenstates of \(S_z\)

\[|\psi\rangle = \psi_0 |0\rangle + \]

Hadamard transform

\[\{ |\pm\rangle \} = \{ |0\rangle \pm |1\rangle \}\]

e.g. eigenstates of \(S_x\)

\[|00\rangle + |11\rangle\]

Bell state

\[\left\{ |p\rangle \right\}_{p \in \mathbb{R}} = \left\{ \int e^{ipq} |q\rangle \right\}_{p \in \mathbb{R}} dq\]

e.g. eigenstates of \(P\): momentum / phase quadr.

**Qumodes**

\[\{ |0\rangle_i, |1\rangle_i \}\]

e.g. eigenstates of \(S_z\)

\[|\psi_i\rangle = \psi_{0i} |0\rangle_i + \]

\[|00\rangle_i + |11\rangle_i\]

Bell state

\[\left\{ |p\rangle_i \right\}_{p \in \mathbb{R}} = \left\{ \int e^{ipq} |q\rangle_i \right\}_{p \in \mathbb{R}} dq\]

e.g. eigenstates of \(P\): momentum / phase quadr.

**EPR state**

\[\int |q\rangle_1 |q\rangle_2 dq\]

\[Q = \frac{1}{\sqrt{2}}(a + a^\dagger)\]

\[P = \frac{i}{\sqrt{2}}(a^\dagger - a)\]

\[|Q, P\rangle = i\]

\[\Delta Q \Delta P = \frac{1}{2}\]

---

Discrete vs. continuous variables, qubits vs. qumodes

\[ Q = \frac{1}{\sqrt{2}} (a + a^\dagger) \]
\[ P = \frac{i}{\sqrt{2}} (a^\dagger - a) \]
\[ [Q, P] = i \]
\[ \Delta Q \Delta P = \frac{1}{2} \]

Because CV SCALE
Gaussian quantum optics (EM fields)
A good starting point: the quantum optical frequency comb

The eigenmodes of a cavity form a large ensemble of classically coherent modes

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)
(10^6 modes oscillating in phase)

Why not turn the QOFC into a quantum computer?
The eigenmodes of a cavity form a large ensemble of classically coherent modes.

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)
(10^6 modes oscillating in phase)

Why not turn the QOFC into a quantum computer?
The eigenmodes of a cavity form a large ensemble of classically coherent modes

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)
(10⁶ modes oscillating in phase)

Why not turn the QOFC into a quantum computer?
A good starting point: the quantum optical frequency comb

The eigenmodes of a cavity form a large ensemble of classically coherent modes

Carrier-envelope-phase locked mode-locked laser = \textit{optical frequency comb (OFC)}
(10^6 modes oscillating in phase)

Why not turn the QOFC into a quantum computer?
The eigenmodes of a cavity form a large ensemble of classically coherent modes.

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC) (10^6 modes oscillating in phase)

Why not turn the QOFC into a quantum computer?
A good starting point: the quantum optical frequency comb

The eigenmodes of a cavity form a large ensemble of classically coherent modes

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)
(10^6 modes oscillating in phase)

Why not turn the QOFC into a quantum computer?
The eigenmodes of a cavity form a large ensemble of classically coherent modes.

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC) (10^6 modes oscillating in phase)

Why not turn the QOFC into a quantum computer?
The eigenmodes of a cavity form a large ensemble of classically coherent modes.

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC) (10^6 modes oscillating in phase).

Why not turn the QOFC into a quantum computer?
The eigenmodes of a cavity form a large ensemble of classically coherent modes.

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC) (10^6 modes oscillating in phase)

Why not turn the QOFC into a quantum computer?
A good starting point: the quantum optical frequency comb

The eigenmodes of a cavity form a large ensemble of classically coherent modes

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)  
(10^6 modes oscillating in phase)

Why not turn the QOFC into a quantum computer?
A good starting point: the quantum optical frequency comb

The eigenmodes of a cavity form a large ensemble of classically coherent modes

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)
(10^6 modes oscillating in phase)

Why not turn the QOFC into a quantum computer?
A good starting point: the quantum optical frequency comb

The eigenmodes of a cavity form a large ensemble of classically coherent modes

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)
(10⁶ modes oscillating in phase)

Why not turn the QOFC into a quantum computer?

Multipartite cluster entanglement in one fell swoop:
a top-down, large-scale quantum register
Multipartite continuous-variable entanglement from concurrent nonlinearities

Olivier Pfister, Sheng Feng, Gregory Jennings, Raphael Pooser, and Daruo Xie

Bright tripartite entanglement in triply concurrent parametric oscillation

A. S. Bradley and M. K. Olsen

ARC Centre of Excellence for Quantum-Atom Optics, School of Physical Sciences,
University of Queensland, Brisbane, Queensland 4072, Australia

O. Pfister and R. C. Pooser
One-Way Quantum Computing in the Optical Frequency Comb

Nicolas C. Menicucci,1,2 Steven T. Flammia,3 and Olivier Pfister4

15 pump modes into YZY, ZZZ, ZYY

- + + - + - + - + + - - + + - - - + - - + - -
Parallel Generation of Quadripartite Cluster Entanglement in the Optical Frequency Comb

Matthew Pysher,1 Yoshichika Miwa,2 Reihaneh Shahrokhshahi,1 Russell Bloomer,1 and Olivier Pfister1,*
Experimental Realization of Multipartite Entanglement of 60 Modes of a Quantum Optical Frequency Comb

Moran Chen, 1 Nicolas C. Menicucci, 2, 7 and Olivier Pfister 1, 7

1 Department of Physics, University of Virginia, Charlottesville, Virginia 22903, USA
2 School of Physics, The University of Sydney, Sydney, New South Wales 2006, Australia
Experimental setup

- Laser1
- Laser2
- Laser3
- PZT
- PLL
- OPO
- PPKTP
- Pump Beams
- PDH Lock
- Entanglement Verification

Equations:
1. $\Omega = (n + \frac{1}{2})\Delta \omega$
2. $2m\Delta \omega$
3. $\omega_{off} \pm 70 \text{ MHz}$
4. $9.2 \text{ MHz}$

Components:
- AOM
- EOM
- HWP
- PBS
- LO
- SA
- PDH Lock
- Flip Mirror
- Cavity
Interfering quantum combs

Interference between identical frequencies of different combs

~$10^4$ modes long

Interfering quantum combs

Interference between identical frequencies of different combs

Chen, Menicucci, and Pfister, Experimental realization of multipartite entanglement of 60 modes of a quantum optical frequency comb, PRL 112, 120505 (2014)
Interfering quantum combs

Interference between identical frequencies of different combs


~10⁴ modes long
Interfering quantum combs

Interference between identical frequencies of different combs

$\sim 10^4$ modes long


Interfering quantum combs

Interference between identical frequencies of different combs


Wavelength-multiplexed quantum networks with ultrafast frequency combs

Jonathan Roslund, Renné Medeiros de Araújo, Shifeng Jiang, Claude Fabre and Nicolas Treps*
Hexapartite Entanglement in an above-Threshold Optical Parametric Oscillator

F. A. S. Barbosa, A. S. Coelho, L. F. Muñoz-Martínez, L. Ortiz-Gutiérrez,
A. S. Villar, P. Nussenzveig, and M. Martinelli

PHYSICAL REVIEW LETTERS 121, 073601 (2018)
Temporal-mode continuous-variable cluster states using linear optics

Nicolas C. Menicucci

Akira Furusawa’s group: sequential entanglement of 10^4 qumodes (2 at a time)

“Dual-rail quantum wire”
Akira Furusawa’s group: *sequential* entanglement of $10^4$ qumodes (2 at a time)
**Quantum Computing**

Generation of time-domain-multiplexed two-dimensional cluster state

Warit Asavanant¹, Yu Shiozawa², Shota Yokoyama³, Baramee Charoenombutamon³, Hiroki Emura¹, Rafael N. Alexander¹, Shuntaro Takeda¹,⁴, Jun-ichi Yoshikawa¹, Nicolas C. Menicucci², Hidehiro Yonezawa², Akira Furusawa¹*  

18 October 2019

---

**Quantum Computing**

Deterministic generation of a two-dimensional cluster state

Mikkel V. Larsen*, Xue Shi Guo, Casner R. Breum, Jonas S. Neerzaad-Nielsen, Ulrik L. Andersen*  

18 October 2019

---

**Diagram:**

- Squeezed lights → Square-shaped cluster states → 2-dimensional cluster state

---

QSS#13 07/16/2020
3D states (and beyond) just around the corner are
3D states (and beyond) just around the corner are

WHY DID STAR WARS EPISODES 4,5,6 COME BEFORE 1,2,3?

BECAUSE IN CHARGE OF PLANNING, YODA WAS
3D states (and beyond) just around the corner are
3D states (and beyond) just around the corner are
The generation of sophisticated, QC-universal cluster states on a very large scale (thousands of qumodes) is EASY over continuous variables and is also highly compatible with an integrated optics approach. (Stay tuned.)

O. Pfister, *Continuous-variable quantum computing in the quantum optical frequency comb*, Journal of Physics B: Atomic, Molecular, and Optical Physics 53, 012001 (2020); invited topical review.
Non-Gaussian quantum optics (photons)
An experimentally accessible non-Gaussian operation: photon-number detection

Sae Woo Nam
(NIST)

Aaron Miller
(Quantum Opus)
An experimentally accessible non-Gaussian operation: photon-number detection

POVM set $= \{ n\  |\ n \} \ n = 0, \ldots, n_{\text{max}}$ (ideally!)

Superconducting transition-edge sensor

Physics Today 71, 8, 28 (2018)
An experimentally accessible non-Gaussian operation: photon-number detection

POVM set $\equiv \{|n\rangle \langle n|\}_{n=0,\ldots,n_{\text{max}}}$ (ideally!)

Superconducting transition-edge sensor

Physics Today 71, 8, 28 (2018)
POVM set $= \{ |n\rangle \langle n| \}_{n=0, \ldots, n_{\text{max}}}$ (ideally!)

Superconducting transition-edge sensor

Physics Today 71, 8, 28 (2018)
An experimentally accessible non-Gaussian operation: photon-number detection

POVM set $\equiv \{ |n\rangle\langle n| \}_{n=0,\ldots,n_{\text{max}}}$ (ideally!)

Superconducting transition-edge sensor

Physics Today 71, 8, 28 (2018)
Processing TES signal: PNR detection

Low photon flux

High flux

~ 15 μs

~ 6 μs
Processing TES signal: PNR detection
Processing TES signal: PNR detection

- Low photon flux: ~15 μs
- High flux: ~6 μs
TES @ PfisterLabs
Quantum tomography with photon counting

S. Wallentowitz & W. Vogel, PRA (1996)

WIGNER FUNCTION

\[ W(q, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ipy} \langle q - \frac{y}{2} | \rho | q + \frac{y}{2} \rangle \, dy \]

- Only function whose marginals yield the quantum probability distributions
- Can be NONPOSITIVE (i.e., nonGaussian for a pure state)
Quantum tomography with photon counting

S. Wallentowitz & W. Vogel, PRA (1996)

WIGNER FUNCTION

\[
W(q, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iyp} \left\langle q - \frac{y}{2} \right| \rho \left| q + \frac{y}{2} \right\rangle dy
\]

- Only function whose marginals yield the quantum probability distributions
- Can be NONPOSITIVE (i.e., nonGaussian for a pure state)

\[
W_\rho(\alpha) = \frac{1}{\pi} Tr[\hat{\rho} D(\alpha)(-1)\hat{n} D^\dagger(\alpha)] \quad \alpha := q + ip
\]

\[
W_\rho(\alpha) = \frac{1}{\pi} Tr[D^\dagger(\alpha)\hat{\rho} D(\alpha)(-1)\hat{n}]
\]

- Expectation value of the photon-number parity
- Easily measured directly with photon-number-resolving detection
- Raster scan of phase space by amplitude/phase shifts gives whole \(W(q,p)\)
State-independent quantum state tomography by photon-number-resolving measurements

RAJVEER NEHRA,1,* AYE WIN,1,5 MILLER EATON,1 © REHANEH SHAHROKHSHAHI,1,4 NIRANJAN SRIDHAR,1,3 THOMAS GERRITS,2 ADRIANA LITA,2 SAE WOO NAM,2 and OLIVIER PFISTER1 ©
Nd: YAG 532 nm, FWHM = 1kHz

- PPKTP(YZY)
- Doubly resonance
- Well below

IF : Interference Filter.
FC : Filter cavity
TES : Transition Edge Sensor

Heralded(Signal) Channel
Heralding(idler) Channel

\[ \eta_h = \frac{N_c}{N_i} = 0.58(2) \]

Overall efficiency of the signal path.
Negativity was observed in the raw data without any inference or correcting for losses.
An even better way to do it

Generalized overlap quantum state tomography

Rajveer Nehra,1,* Miller Eaton,1,† Carlos González-Arciniegas,1 M. S. Kim,2,3 Thomas Gerrits,4 Adriana Lita,4 Sae Woo Nam,4 and Olivier Pfister1,‡

- Efficient reconstruction using semidefinite programming
Non-Gaussian resources $\Rightarrow$ quantum error correction

Gottesman-Kitaev-Preskill (GKP) States

Simultaneous eigenstates of
\[ X = e^{-i \hat{P} \alpha}, \quad Z = e^{i \frac{\alpha}{\hbar} \hat{Q}} \]

\[ |0\rangle, \quad |\bar{1}\rangle, \quad |\frac{2\pi}{\alpha}\rangle, \quad |\frac{2\pi}{\alpha}\rangle \]


Feed-forward displacement to correct error

Ancilla GKP states

Non-Gaussian resources $\Rightarrow$ quantum error correction

Gottesman-Kitaev-Preskill (GKP) States

Simultaneous eigenstates of

\[ X = e^{-i\hat{P}x}, \quad Z = e^{i\frac{\pi}{\alpha}\hat{Q}} \]

- $|\tilde{0}\rangle$
- $|\tilde{1}\rangle$
- $|\tilde{0}\rangle + |\tilde{1}\rangle$
- $|\tilde{0}\rangle - |\tilde{1}\rangle$


Feed-forward displacement to correct error


Non-Gaussian resources => quantum error correction

Gottesman-Kitaev-Preskill (GKP) States

Simultaneous eigenstates of

\[ X = e^{-i\hat{P}x}, \quad Z = e^{i\frac{\pi}{\alpha}\hat{Q}} \]

| \( |0\rangle \): | \( |1\rangle \): | \( |0\rangle + |1\rangle \): | \( |0\rangle - |1\rangle \): |
|---|---|---|---|
| \[ 2\alpha \] | \[ 2\alpha \] | \[ 2\pi / \alpha \] | \[ 2\pi / \alpha \] |

Feed-forward displacement to correct error

\[ \tilde{f}(q_0, p_0) \]


Realistic implementation: Delta spikes \( \rightarrow \) peak width \( \Delta \)

\[ \Delta = 0.3 \]

\[ |\tilde{0}\rangle \propto e^{\Delta^2 \hat{N}} \sum_{n_1, n_2 = -\infty}^{\infty} e^{-i n_1 \hat{P} x} e^{i n_2 \hat{Q} x} |\text{vac}\rangle \]

Ancilla GKP states

Cluster state
defines nodes

GKP states

Error corrected

Quantum engineering of GKP states for CV quantum error correction


BIG PICTURE: beating lattice-gauge QCD calculations on classical supercomputers

Quantum Algorithms for Quantum Field Theories
Stephen P. Jordan, 1, 2 Keith S. M. Lee, 2 John Preskill 3
1 JUNE 2012 VOL 336 SCIENCE www.sciencemag.org

PHYSICAL REVIEW A 92, 063825 (2015)

Quantum simulation of quantum field theory using continuous variables
Kevin Marshall, 1 Raphael Pooser, 2, 3 George Siopsis, 3, 4 and Christian Weedbrook 4

\[ A = \langle \text{out}\vert T \exp \left\{ i \int_{-T}^{T} dt [H_{\text{in}}(t) + H_{\text{c.f.}}(t)] \right\} \vert \text{in} \rangle \]

\( \phi \) term

(rotate all beamsplitters by 90°)
Conclusion
Conclusion

The quantum optical frequency comb is a viable platform for universal quantum computing
The quantum optical frequency comb is a viable platform for universal quantum computing

- LARGE-SCALE Gaussian entanglement
- No postselection
- Universal measurement-based QC, strictly equivalent to qubit model
Conclusion

The quantum optical frequency comb is a viable platform for universal quantum computing

- **LARGE-SCALE** Gaussian entanglement
  - No postselection
  - Universal measurement-based QC, strictly equivalent to qubit model

- **FAULT TOLERANT** non-Gaussian technology for universal QC
Conclusion

The quantum optical frequency comb is a viable platform for universal quantum computing

- **LARGE-SCALE** Gaussian entanglement
  - No postselection
  - Universal measurement-based QC, strictly equivalent to qubit model

- **FAULT TOLERANT** non-Gaussian technology for universal QC

- **INTEGRABLE** in photonic circuits
The quantum optical frequency comb is a viable platform for universal quantum computing

- **LARGE-SCALE** Gaussian entanglement
  - No postselection
  - Universal measurement-based QC, strictly equivalent to qubit model

- **FAULT TOLERANT** non-Gaussian technology for universal QC

- **INTEGRABLE** in photonic circuits

- **TRANSLATABLE** to any bosonic mode:
  - microwave cavity photons
  - phonons
  - transverse spatial modes (Hermite- or Laguerre-Gauss)
  - temporal modes
A lesson in research from Ted Hänsch...