Solving computational problems with coupled lasers

Nir Davidson

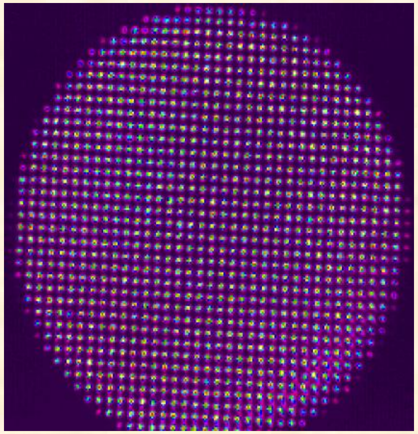
S. Mahler, I. Gershenzon, S. Gadasi, N. Andra, G. Arwas,

(C. Tradonsky, R. Chriki, V. Pal, M. Nixon, E. Ronen, M. Fridman)

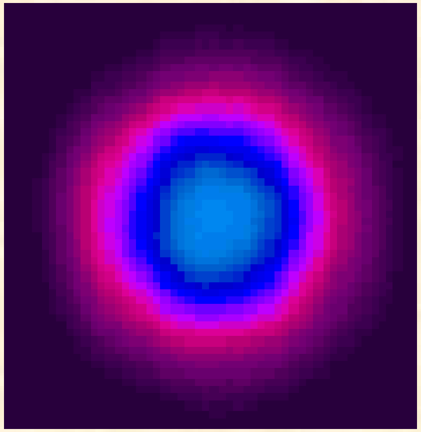
O. Raz, and A. A. Friesem

Weizmann Institute of Science

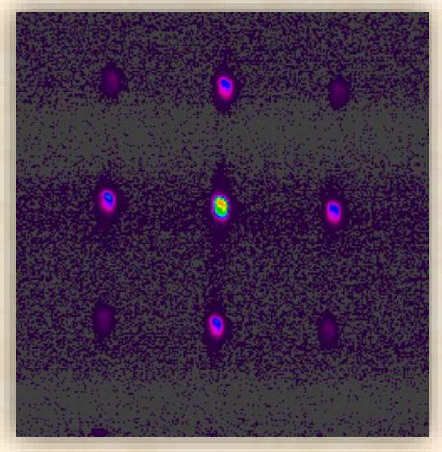
1500 lasers



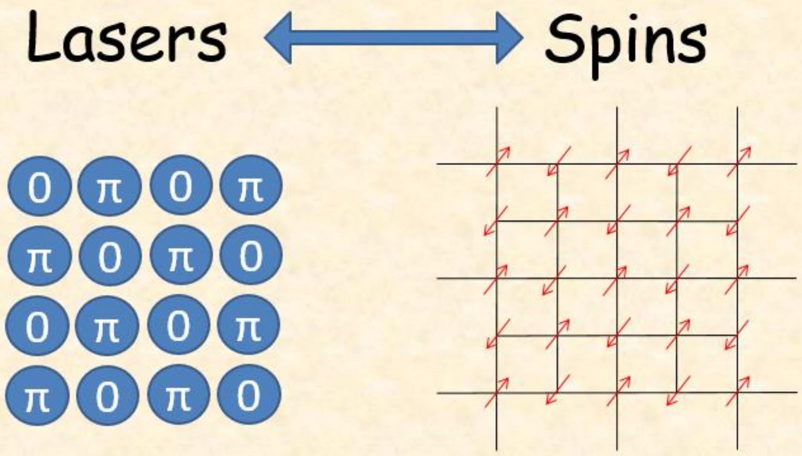
Far-Field
no coupling

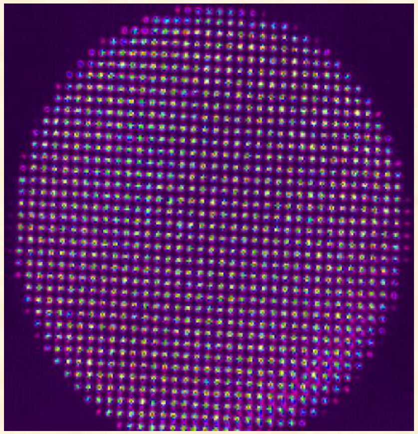


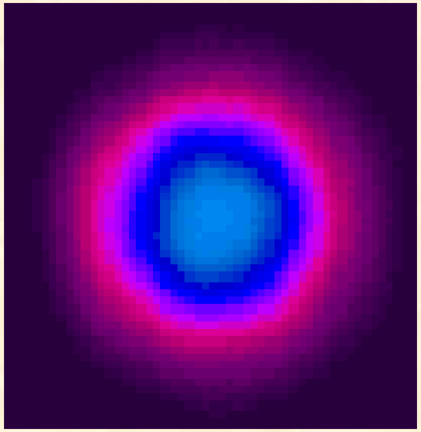
Far-Field
with coupling

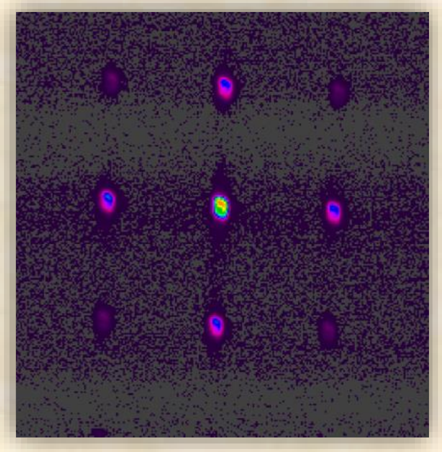


Solving computational problems with coupled lasers

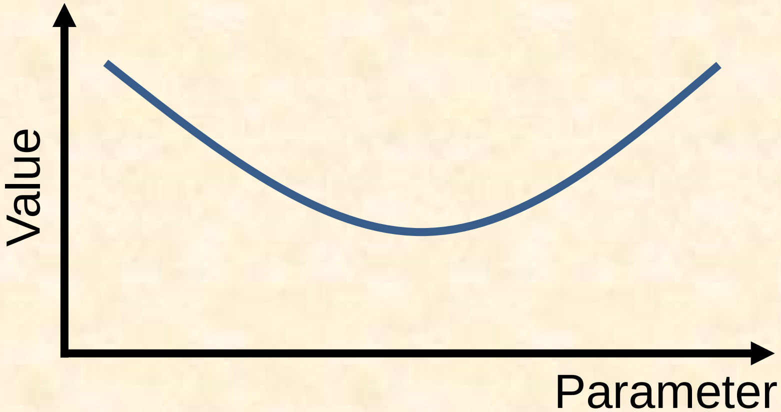


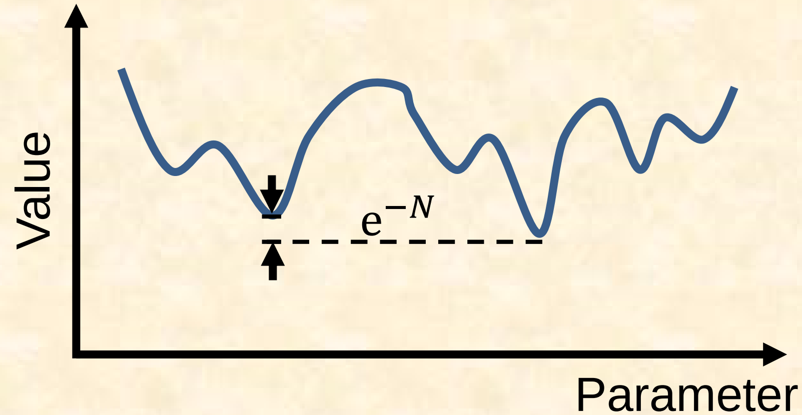






Coherent computing: basic principle

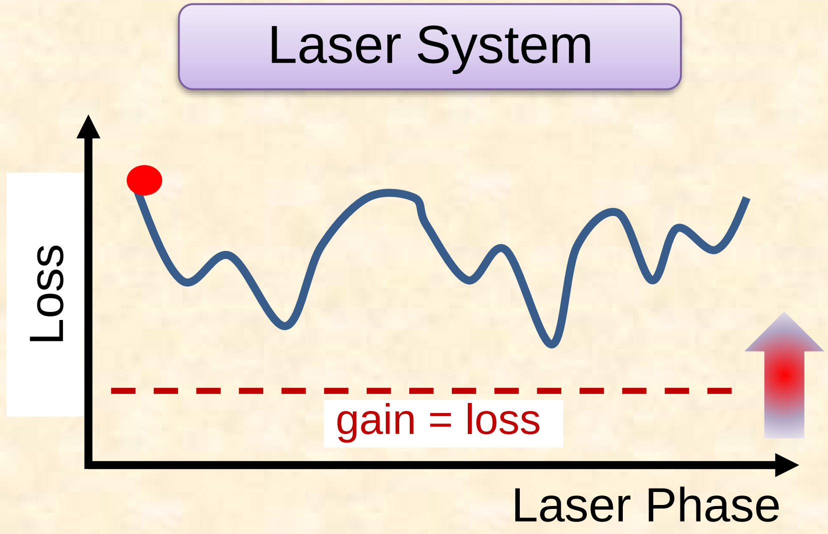




"Easy"

"Hard"

Coherent computing: basic principle



Science

REPORTS

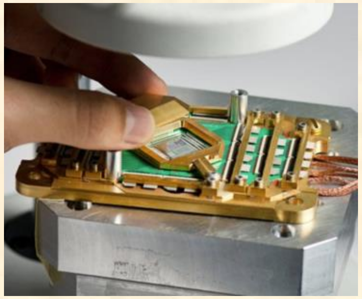
Cite as: P. L. McMahon *et al*., *Science* 10.1126/science.aah5178 (2016).

A fully-programmable 100-spin coherent Ising machine with all-to-all connections

Peter L. McMahon,[1,2*†] Alireza Marandi,[1*†] Yoshitaka Haribara,[2,3,4] Ryan Hamerly,[1] Carsten Langrock,[1] Shuhei Tamate,[2] Takahiro Inagaki,[5] Hiroki Takesue,[5] Shoko Utsunomiya,[2] Kazuyuki Aihara,[3,4] Robert L. Byer,[1] M. M. Fejer,[1] Hideo Mabuchi,[1] Yoshihisa Yamamoto[1,6]

A. Marandi et al, Nature Photon. 8, 937 (2014)

"Annealing" machines (very partial list)

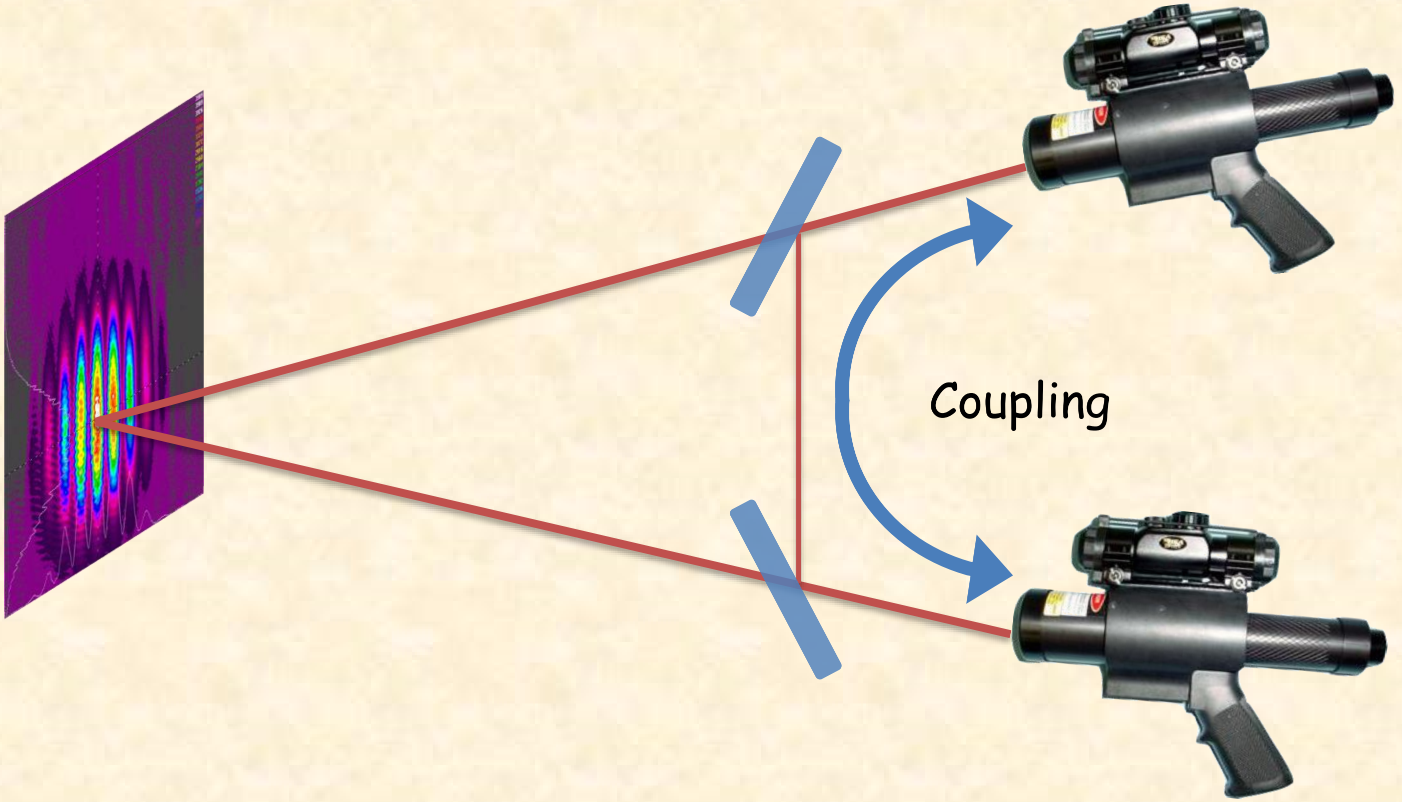
- D-Wave 
- OPO's (Stanford + NTT, Columbia, Cornell, Caltech …)
- Lasers (Weizmann, Light-Solver, CREOL/USC, Tucson, Stanford + Tokyo …)
- Polariton BECs (Southampton, Cambridge, Palaiseau …)
- Photonics (Rome, MIT …)
- Electronics (MIT, Fujitsu, Toshiba, Bar-Ilan …)
- Micro-droplets (Toronto)
- Cold atoms (many places)

Outline

- Simulating XY spins and beyond
- Continuous problems (if time permits)

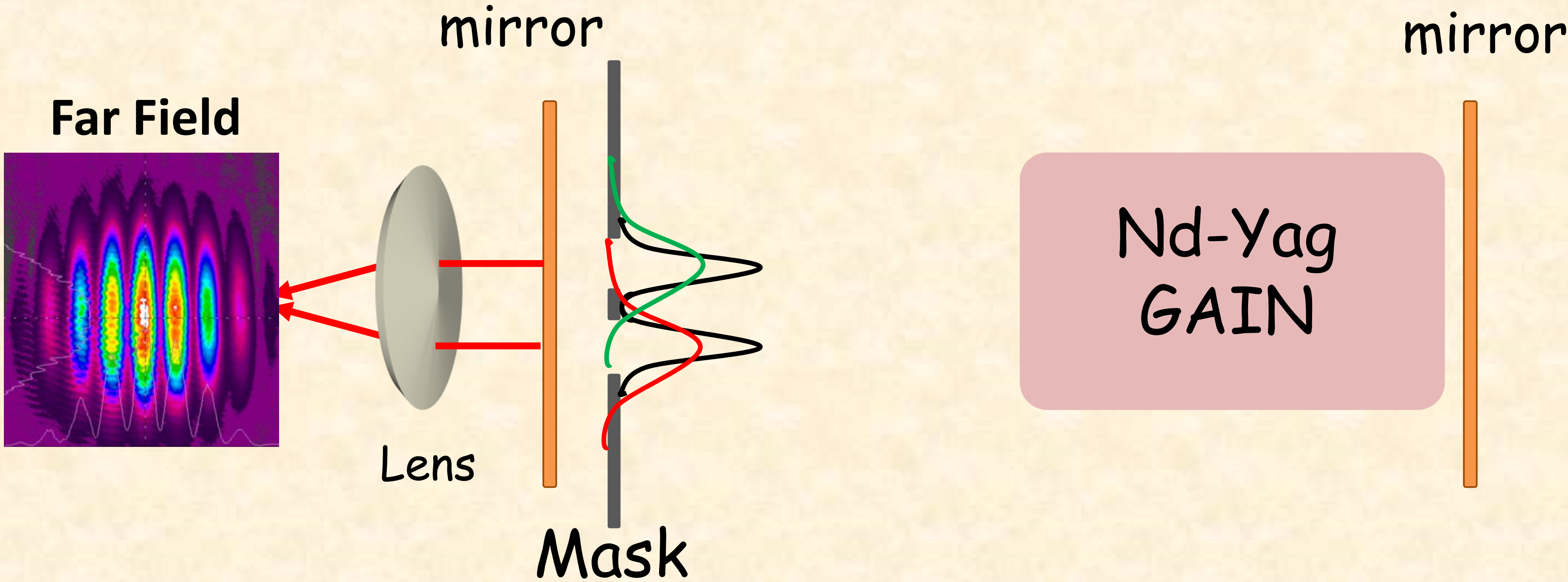
R. Chriki et. al., Nature Physics Review 1, 156 (2019)

What is phase locking ?



$$\Delta\varphi(t) = \varphi_2(t) - \varphi_1(t) = const$$

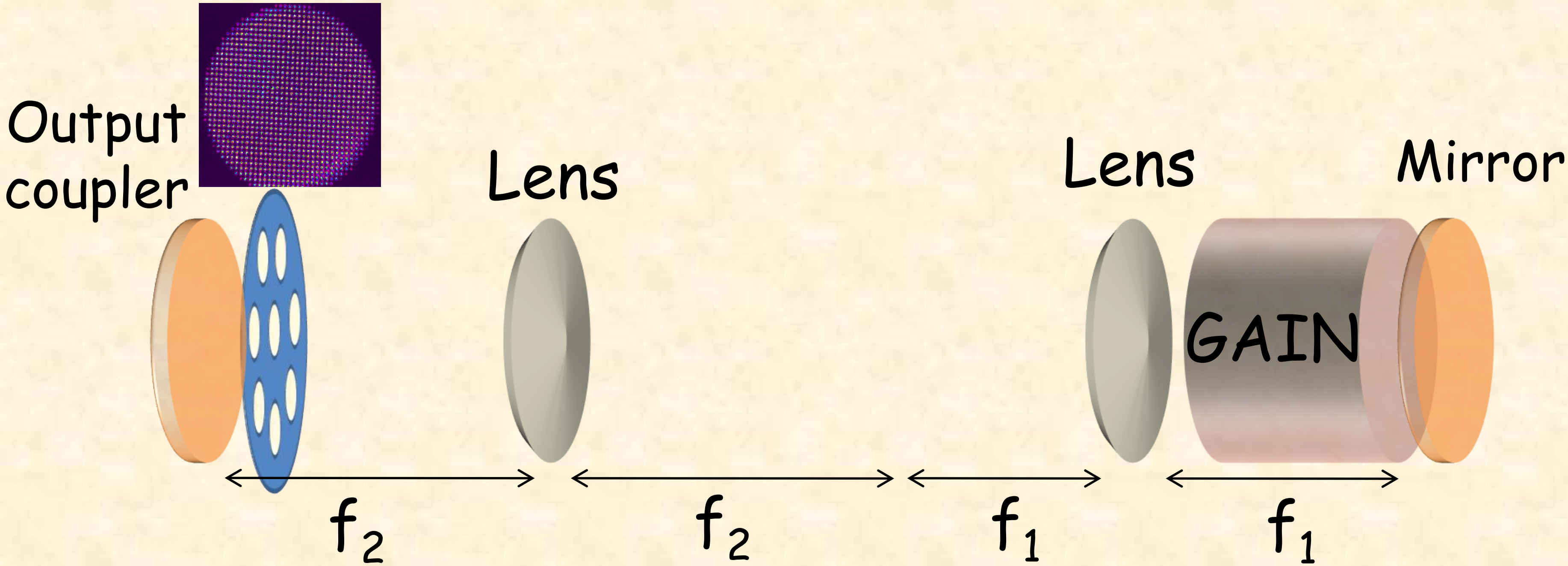
Diffractive coupling



Short range coupling $\kappa_{ij} = \langle E_i | E_j \rangle \cong e^{-\alpha(i-j)^2}$

Dissipative coupling: minimizes losses

Degenerate laser cavity

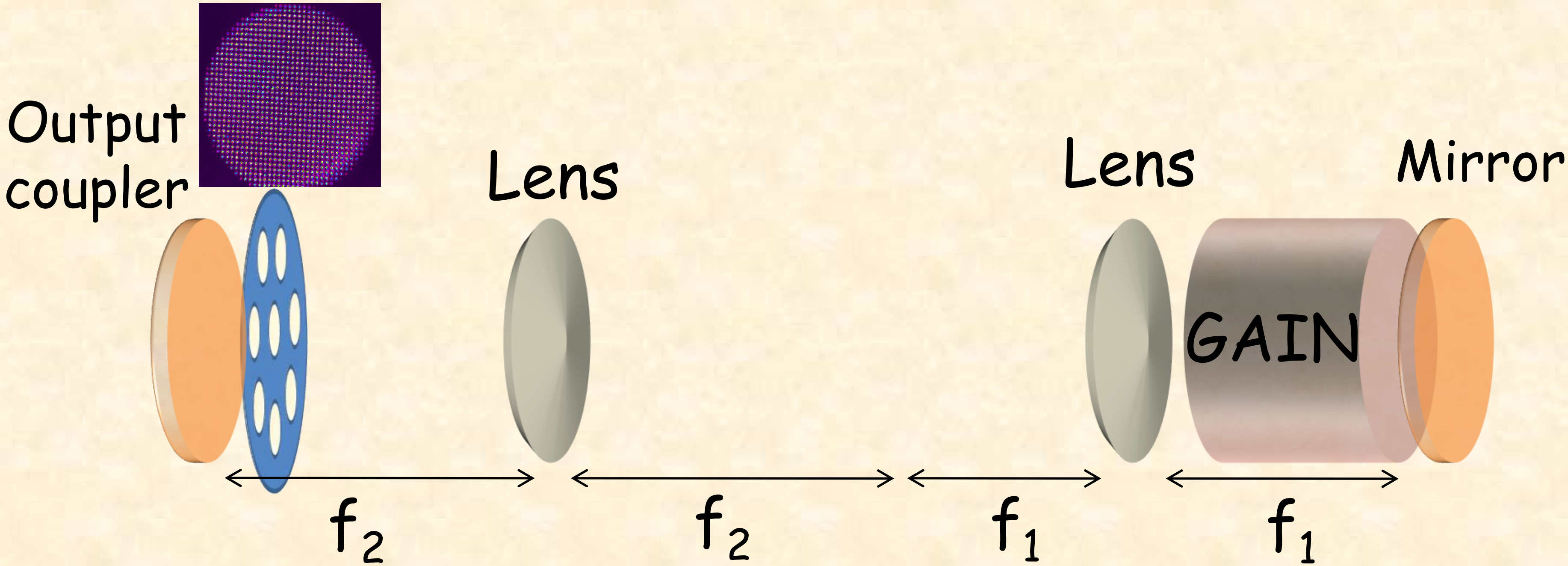


$$E(x,y) \rightarrow E(-x,-y) \rightarrow E(x,y)$$

320,000 degenerate modes (R. Chriki 2019)

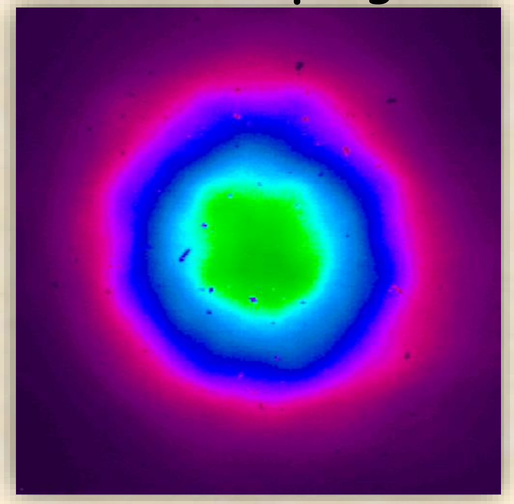
M. Nixon et. al., PRL 106, 223901 (2011); PRL 108, 214101 (2012)

Degenerate cavity: square array

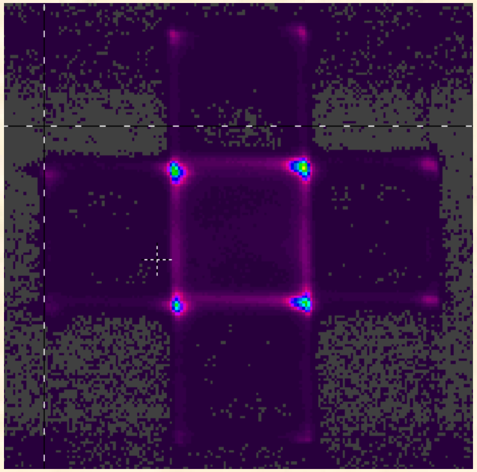


Far Field

No coupling

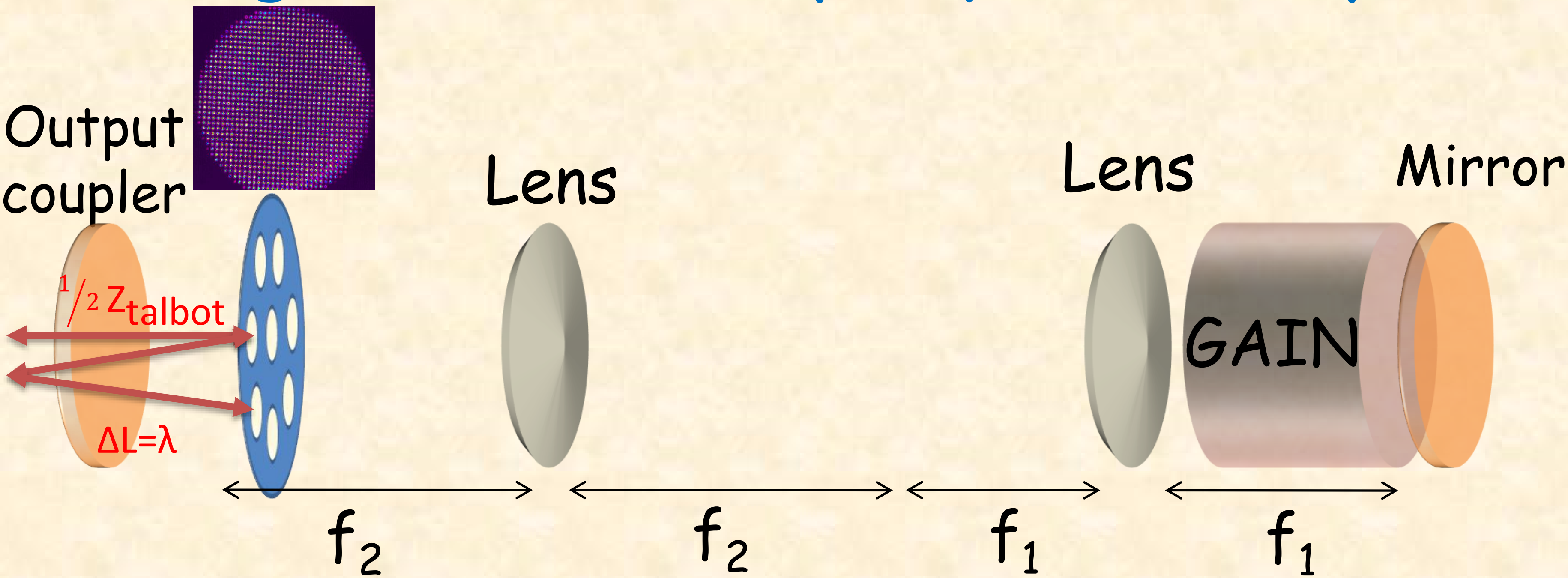


Negative coupling



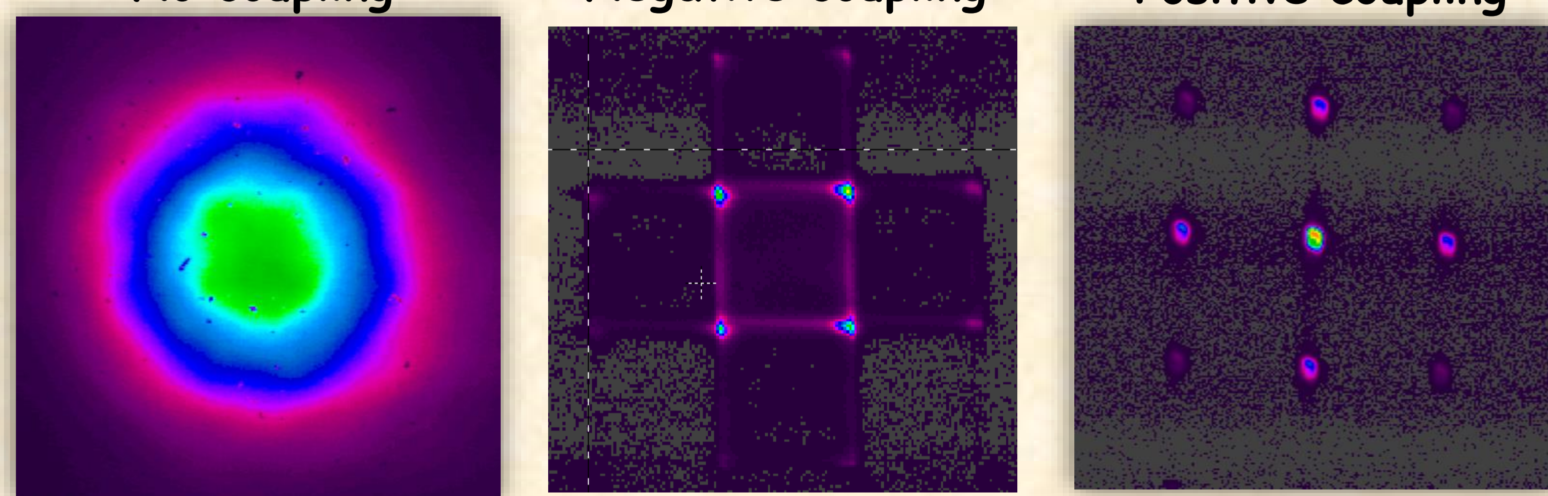
M. Nixon et. al., PRL 106, 223901 (2011); PRL 108, 214101 (2012)

Degenerate cavity: square array



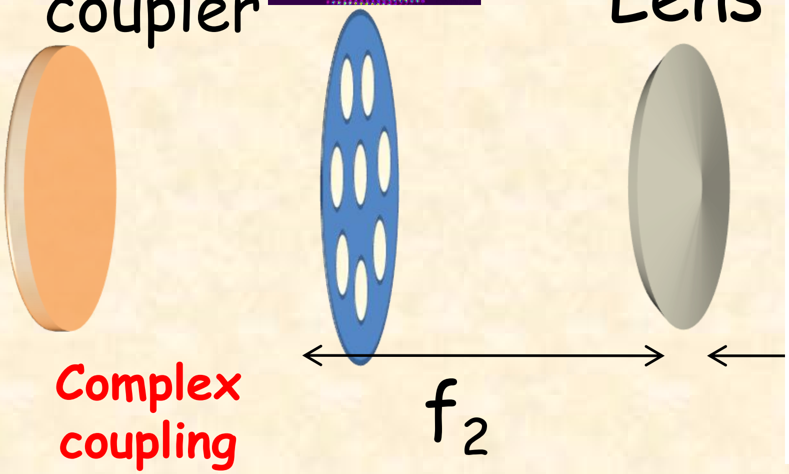
No coupling | Negative coupling | Positive coupling

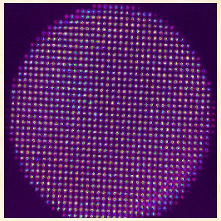
Far Field

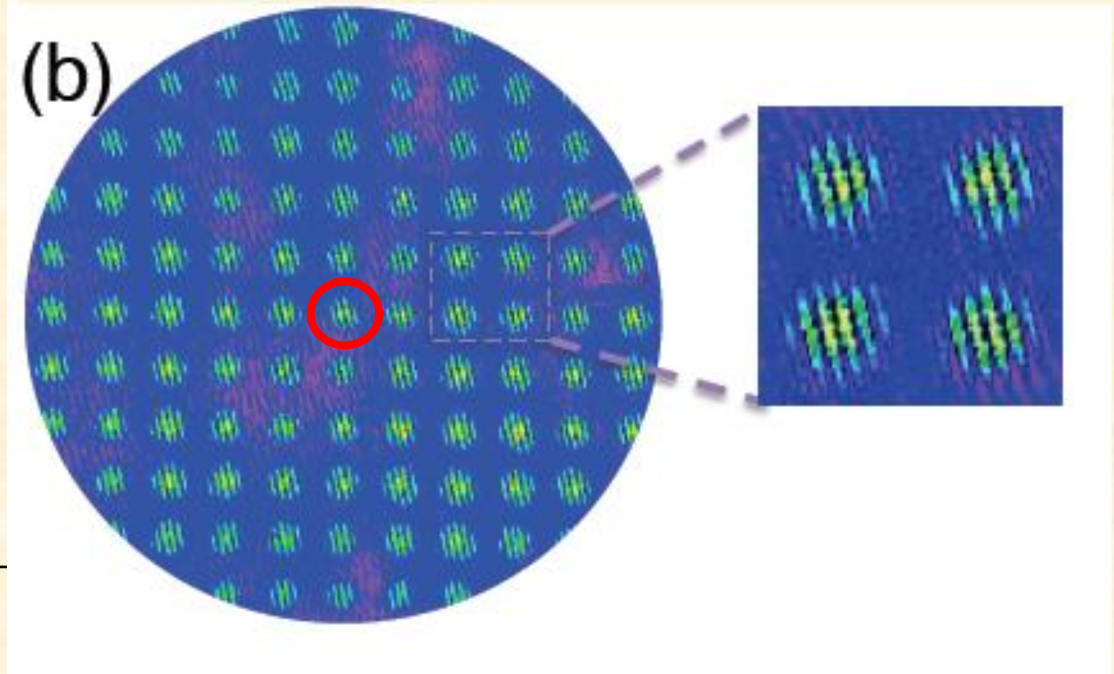


M. Nixon et. al., PRL 106, 223901 (2011); PRL 108, 214101 (2012)

Degenerate cavity: square array

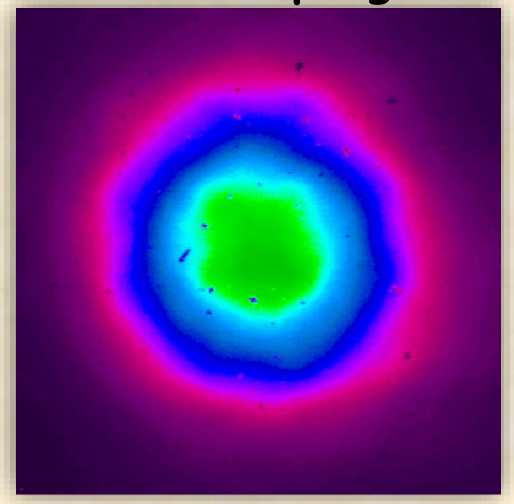




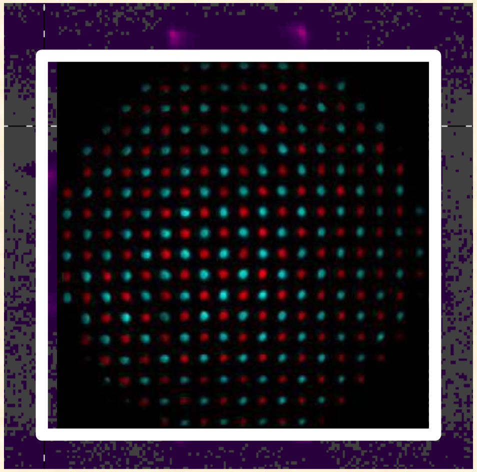


Far Field

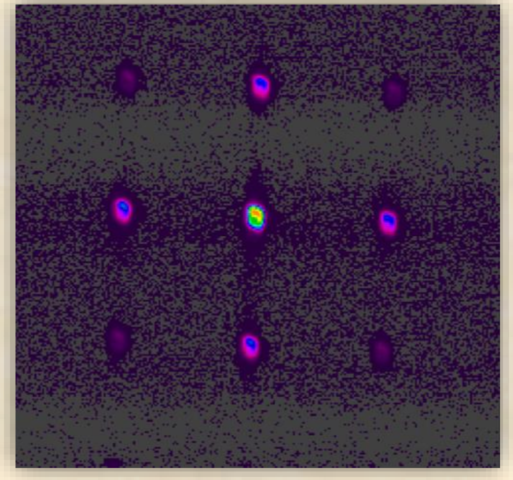
No coupling



Negative coupling



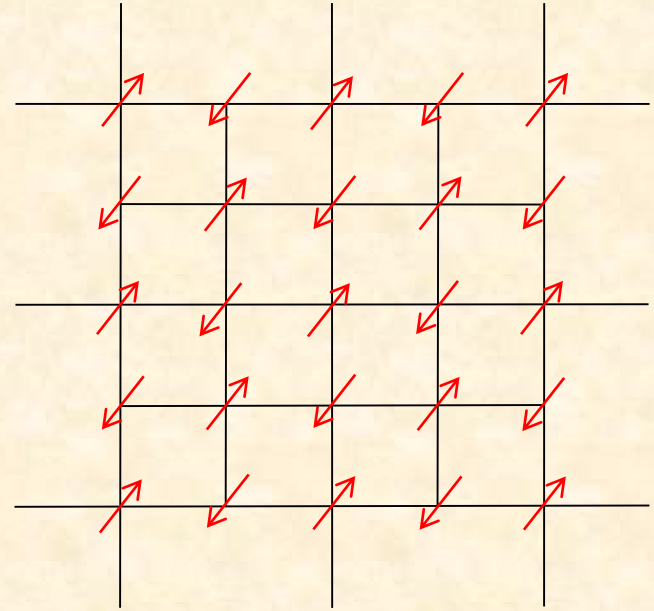
Positive coupling



M. Nixon et. al., PRL 106, 223901 (2011); PRL 108, 214101 (2012)

Lasers ⟷ Spins ?

("freeze" all DOF except phase)



$$\dot{\theta}_i = \Sigma \kappa_{ij} \sin(\theta_j - \theta_i)$$

$$H = \Sigma \kappa_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Kuramoto Model | **Classical XY model**

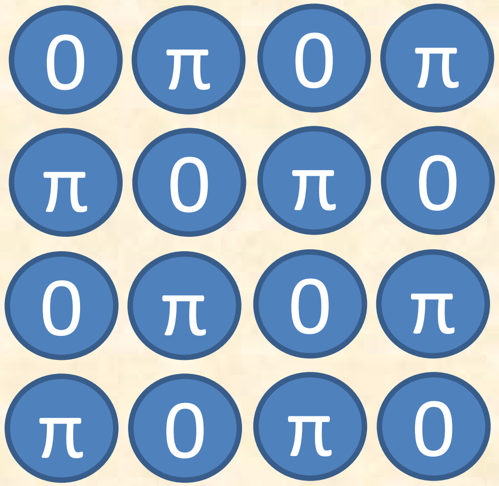
Laser "minimal loss" state ⟷ Kuramoto stable fixed point ⟷ XY ground state

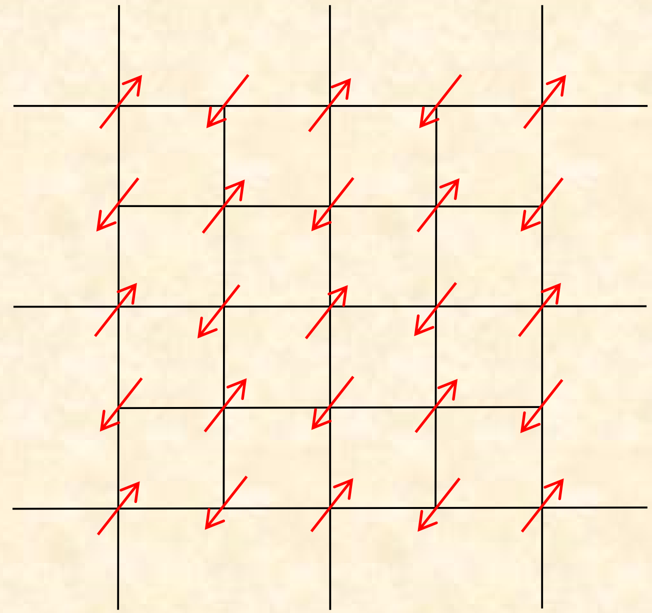
M. Nixon et. al., PRL 110, 184102 (2013)

Lasers ⟷ Spins

?

("freeze" all DOF except phase)





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Kuramoto Model | **Classical XY model**

Laser "minimal loss" state ↮ Kuramoto stable fixed point ⟷ XY ground state

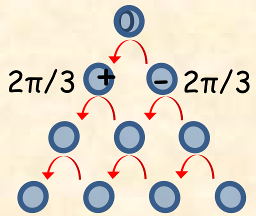
I. Gershenzon et. al., Nanophotonics 20200137 (2020)

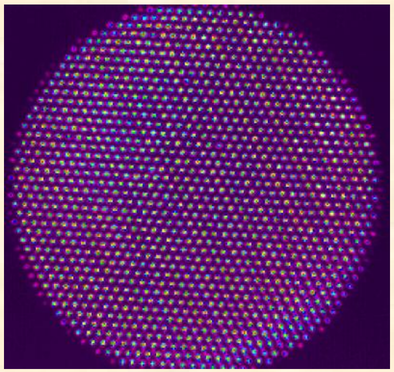
M. Nixon et. al., PRL 110, 184102 (2013)

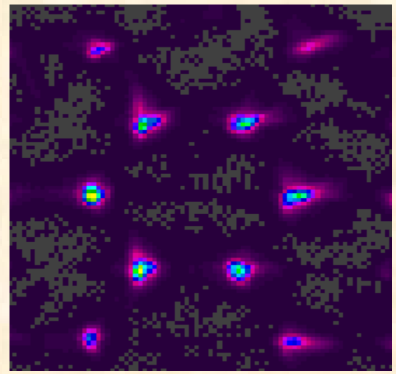
Large arrays (NN negative coupling)

Near Fields | Far Fields

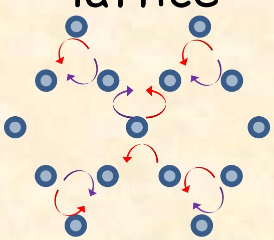
Triangular lattice

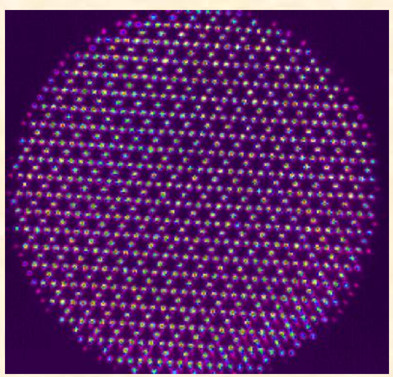


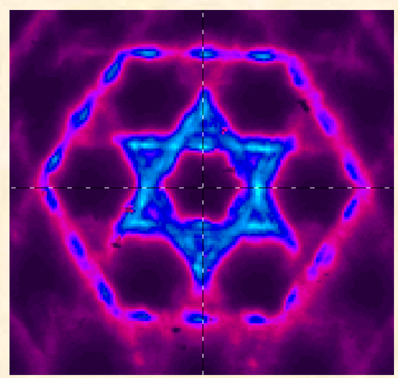


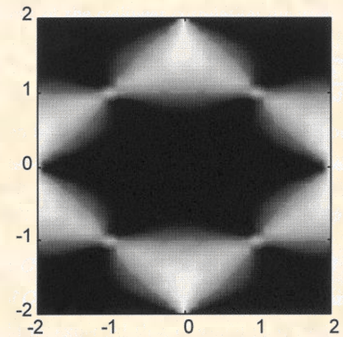


Kagome lattice









Moessner and Chalker
PRB **58** 12049 (1998)

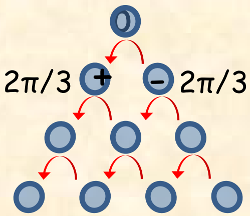
M. Nixon et. al., PRL 110, 184102 (2013); V. Pal et. al., PRR 2, 033008 (2020)

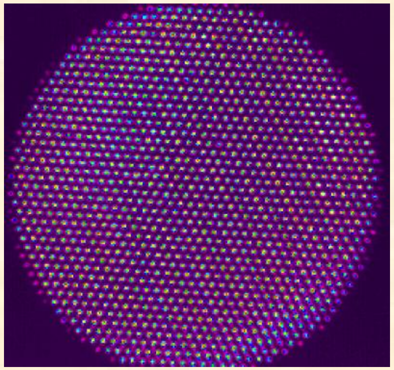
Large arrays (NN negative coupling)

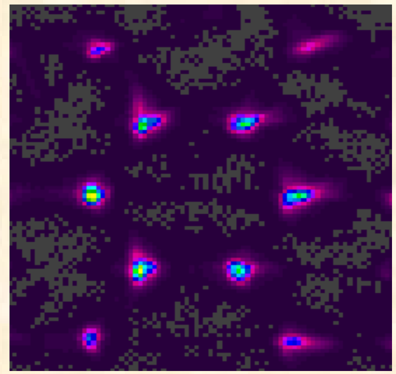
Near Fields

Far Fields

Triangular lattice

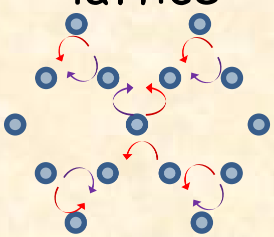


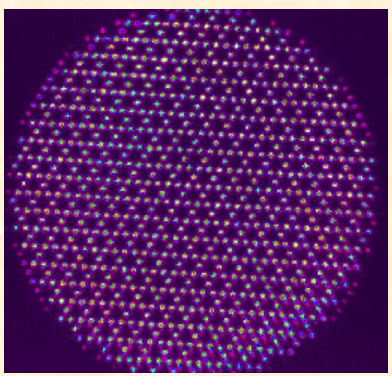


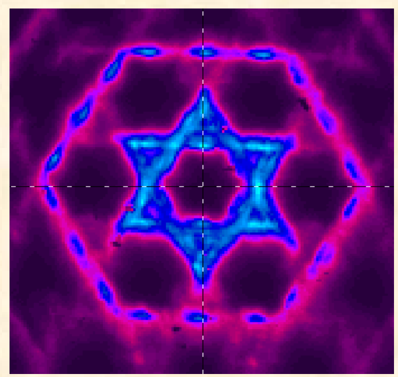


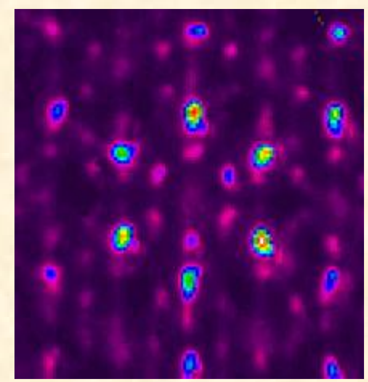
NNN coupling

Kagome lattice



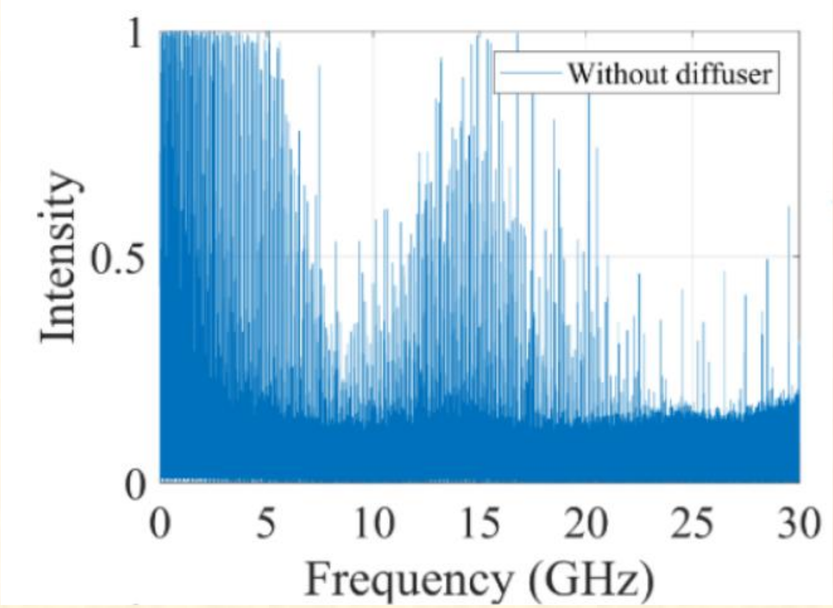






M. Nixon et. al., PRL 110, 184102 (2013); V. Pal et. al., PRR 2, 033008 (2020)

Spatio-temporal modes



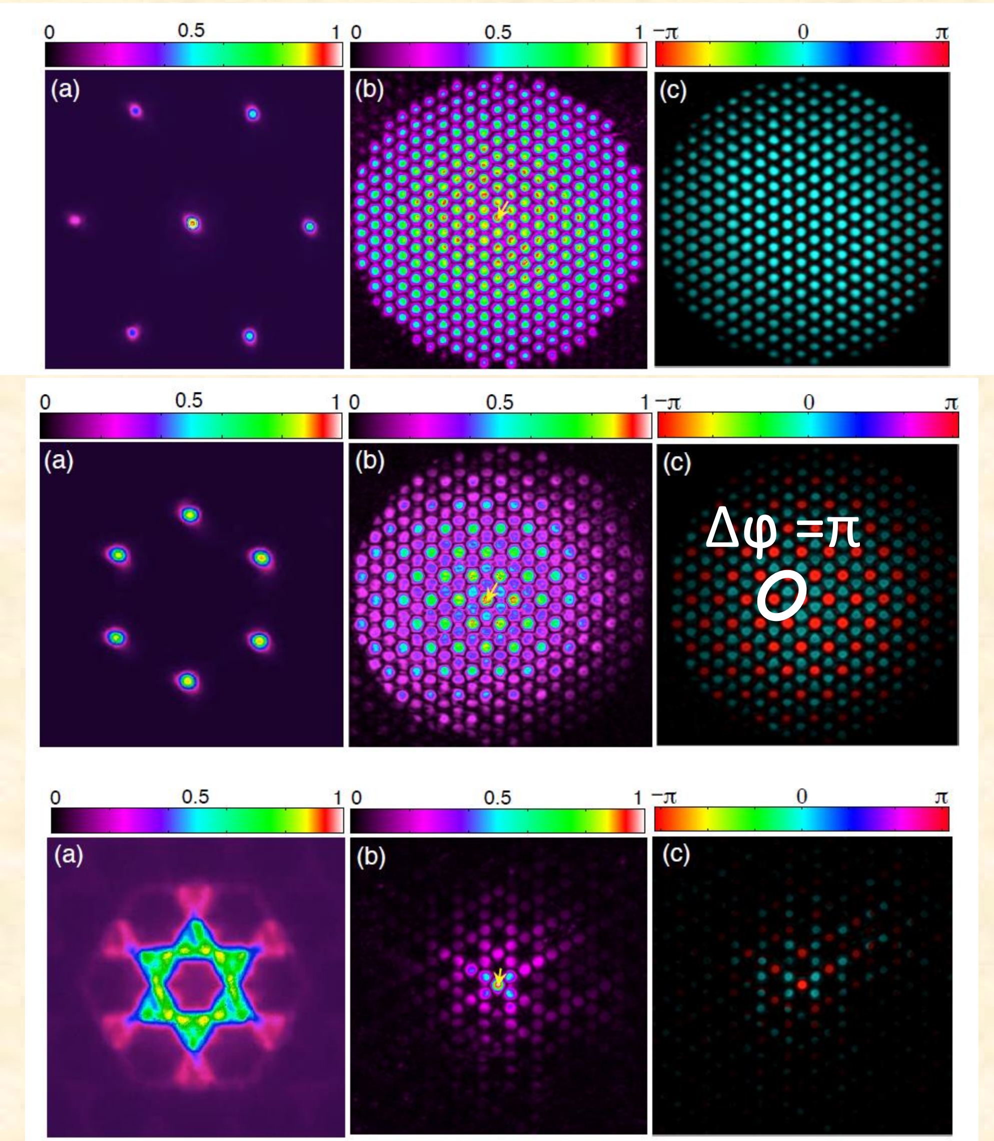
Each of the 1500 lasers has 250 longitudinal modes

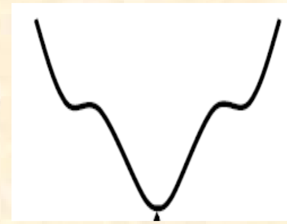
Ensemble of 250 independent experiments

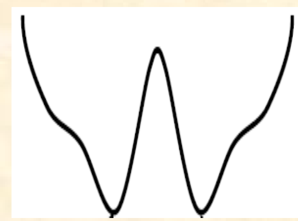
S. Mahler et. al., Nano photonics, in press (2020)

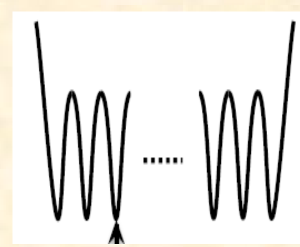
Average ‹cos(φ_{ij})› over 250 longitudinal modes

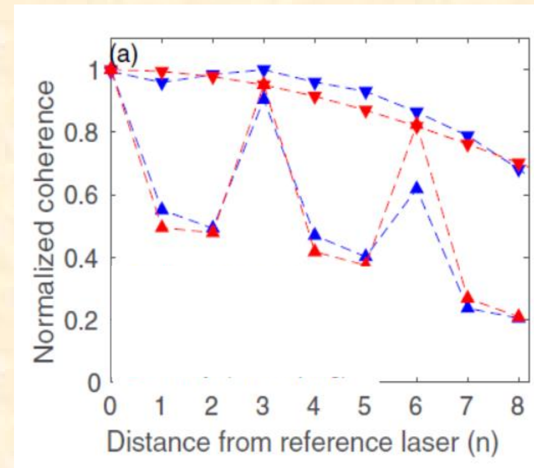
Far Field Coherence

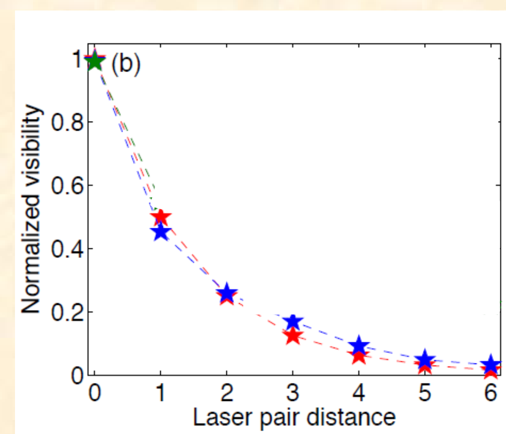












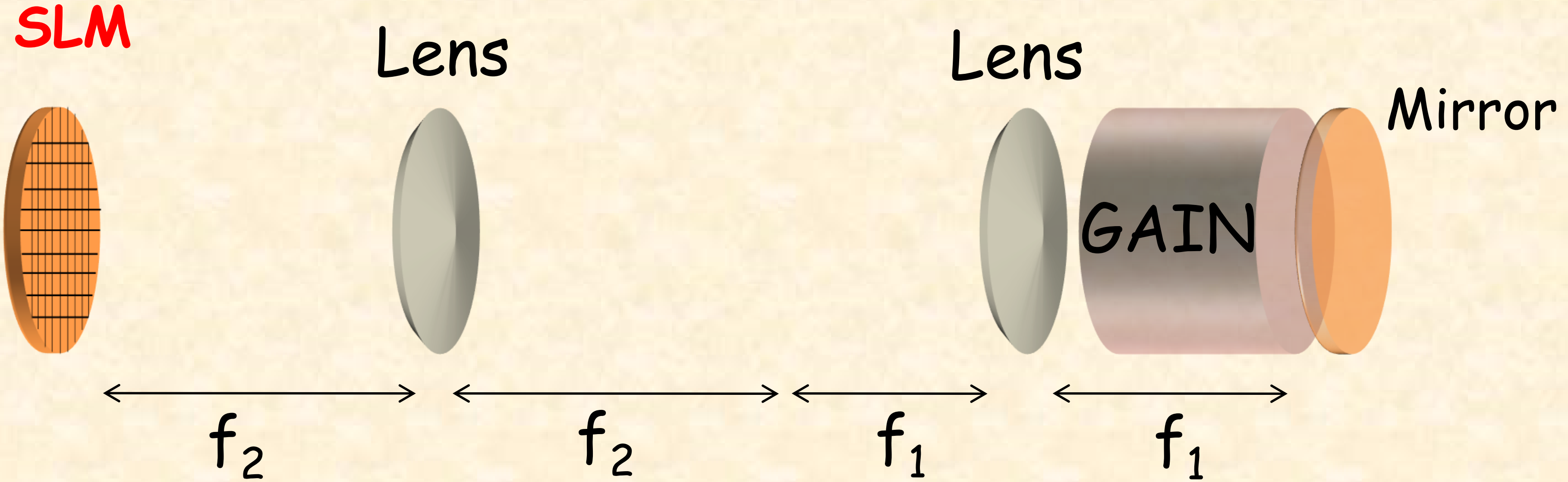
V. Pal et. al., PRR 2, 033008 (2020)

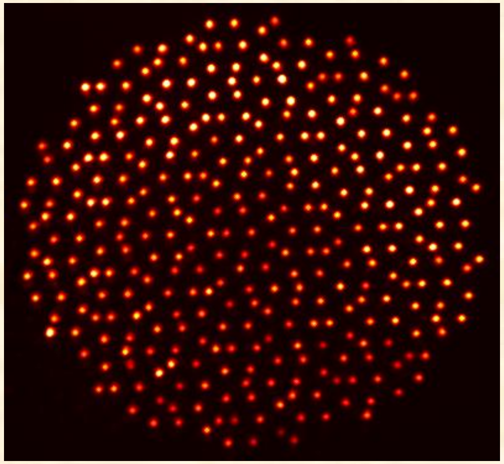
Synchronization in human networks

S. Shahal et. al. , Nature Comm. 11, 3854 (2020)

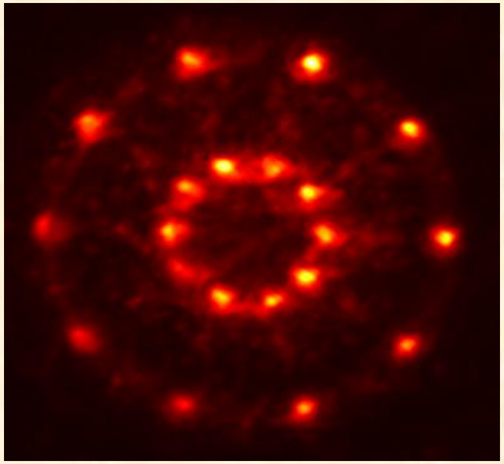
Questions so far?

Digital laser (amplitude)





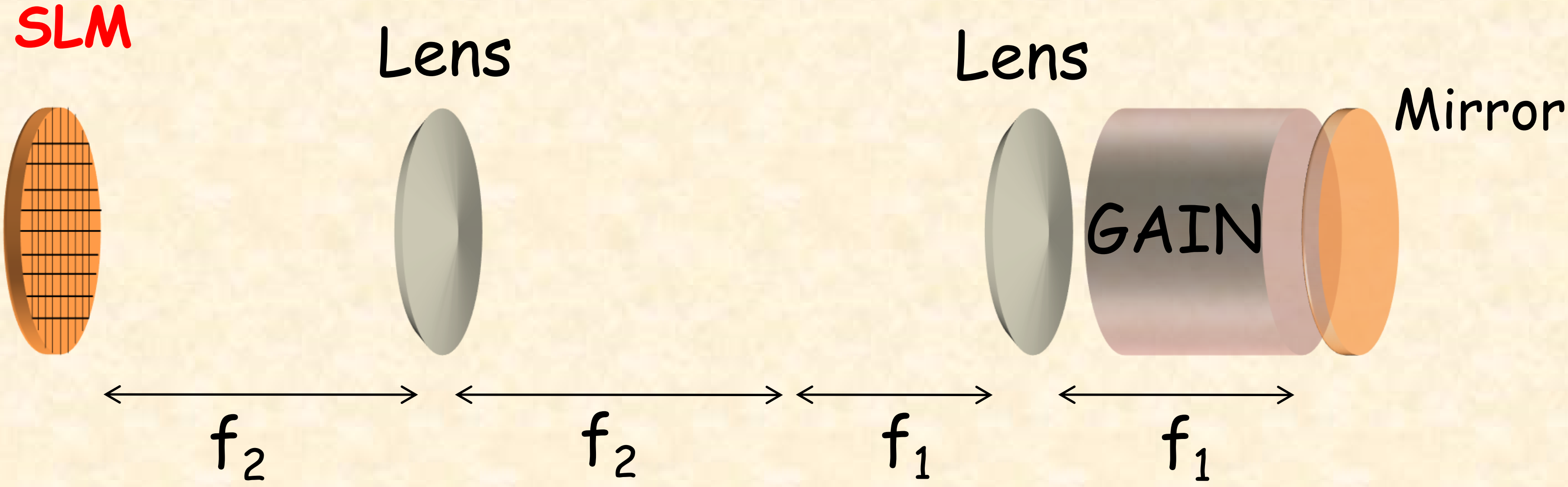
Near Field

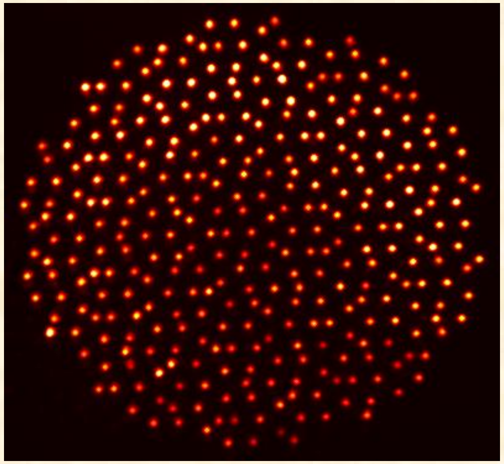


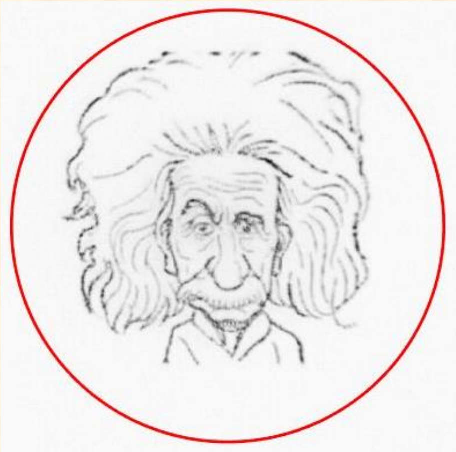
Far Field

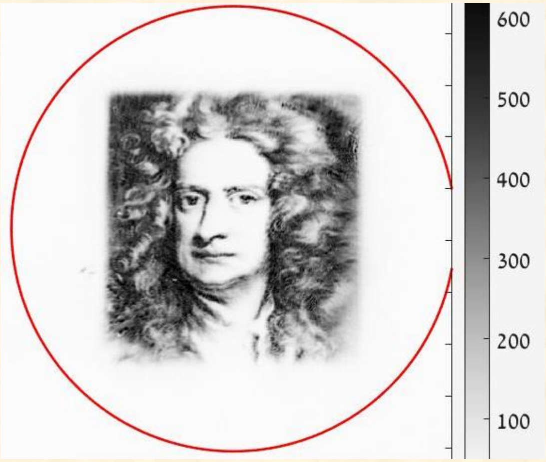
C. Tradonsky et. al. unpublished

Digital laser (amplitude)



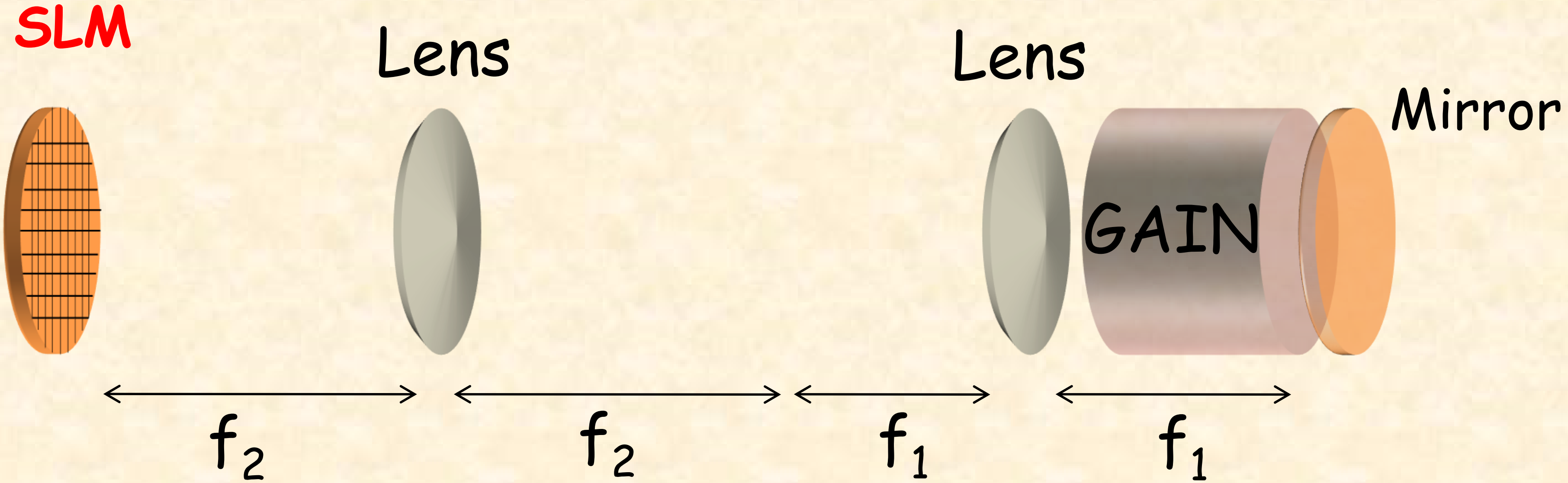






C. Tradonsky et. al. unpublished

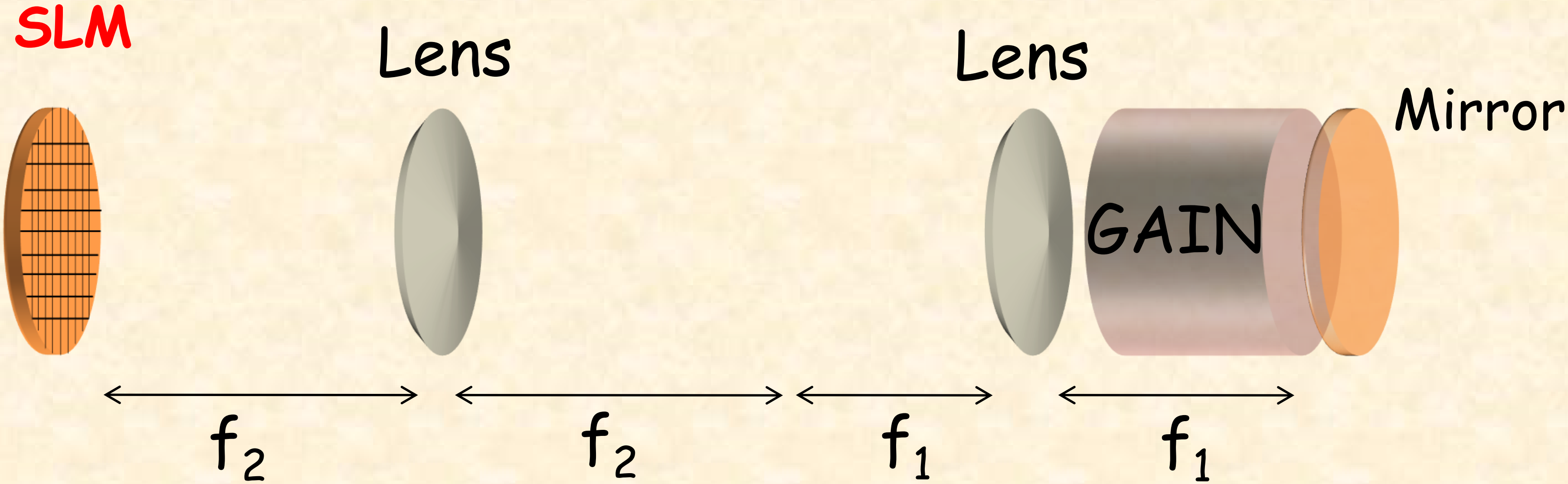
Adding disorder (phase)

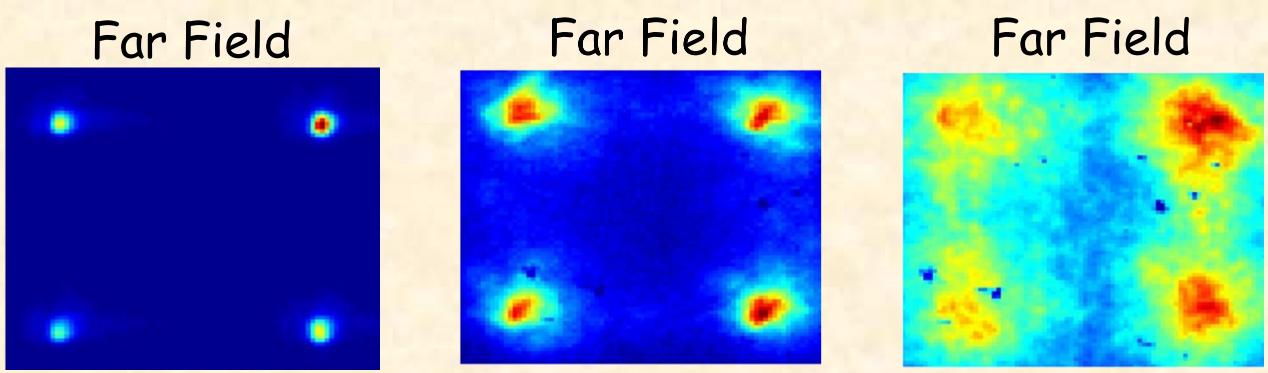


$$\dot{\theta}_i = \Omega_i + \Sigma \kappa_{ij} \sin(\theta_j - \theta_i)$$

E. Ronen et. al. unpublished

Adding disorder

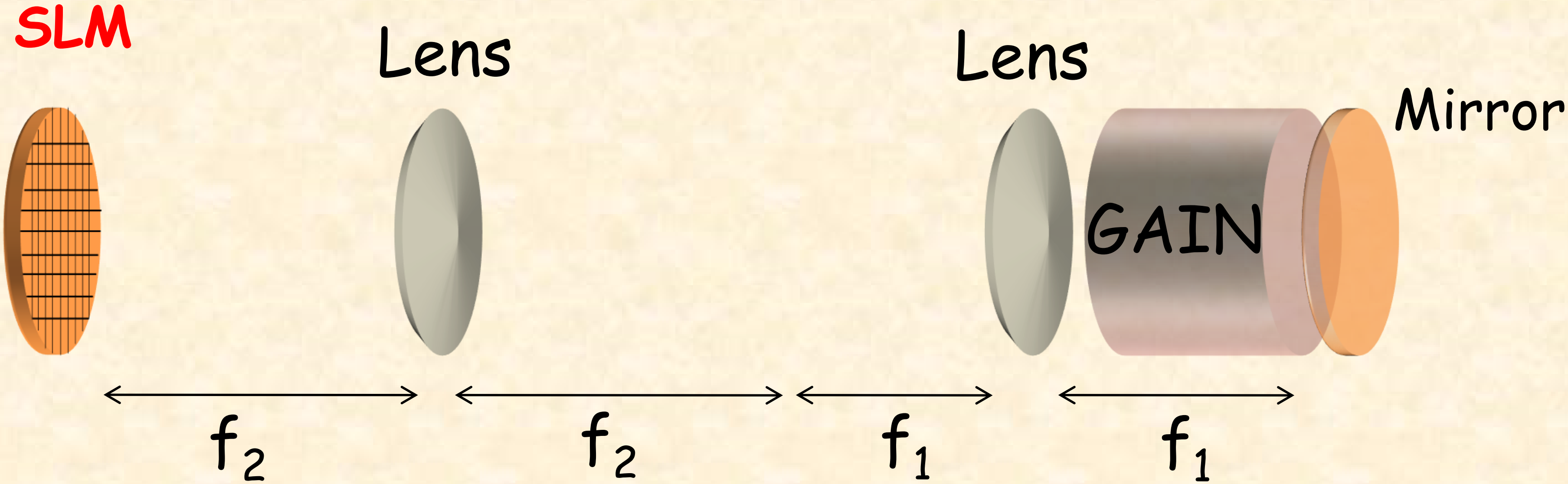


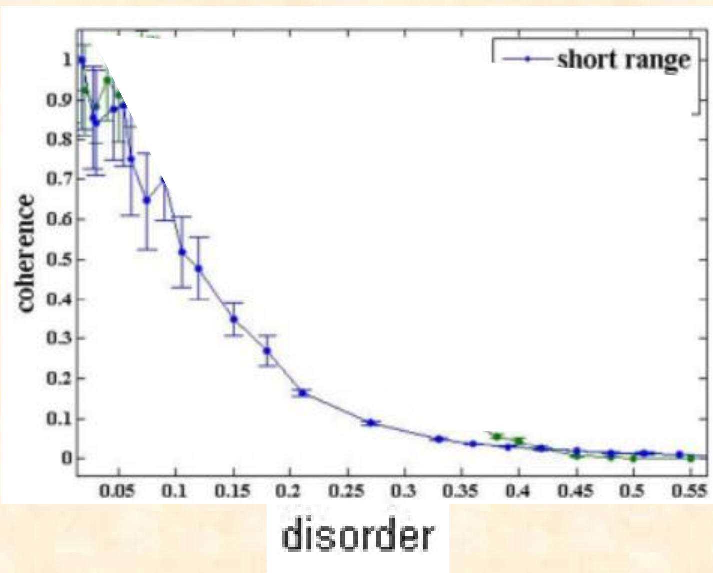


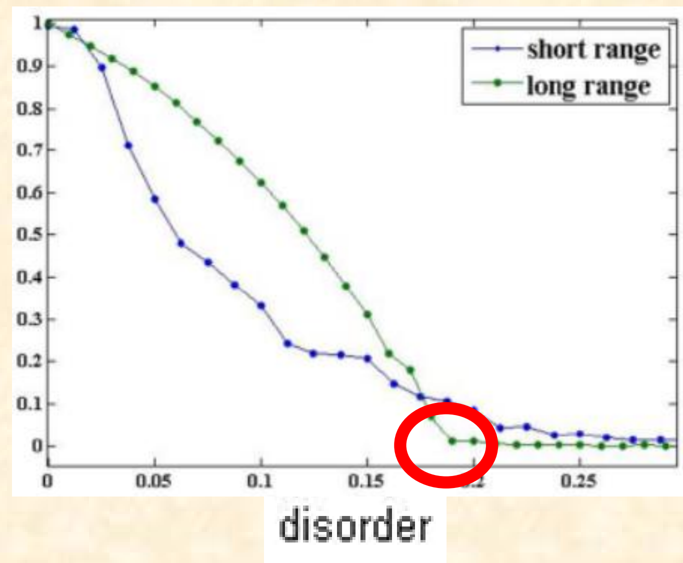
$\langle \varphi \rangle_{rms} = 0$ $\qquad$ $\langle \varphi \rangle_{rms} = \frac{\pi}{10}$ $\qquad$ $\langle \varphi \rangle_{rms} = \frac{\pi}{5}$

E. Ronen et. al. unpublished

Adding disorder



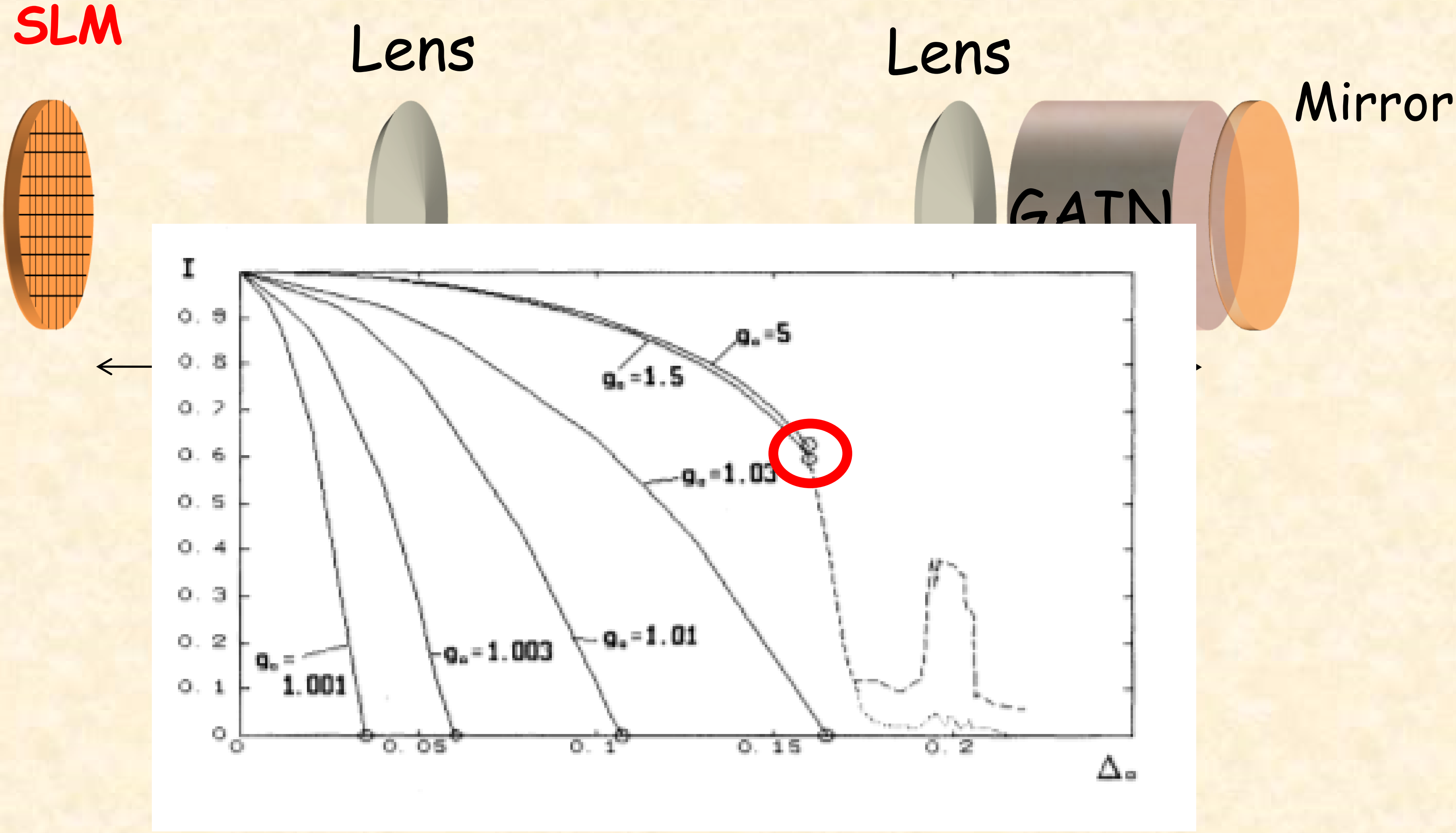




E. Ronen et. al. unpublished

Kuramoto, Arecchi, Strogatz, McCall, …

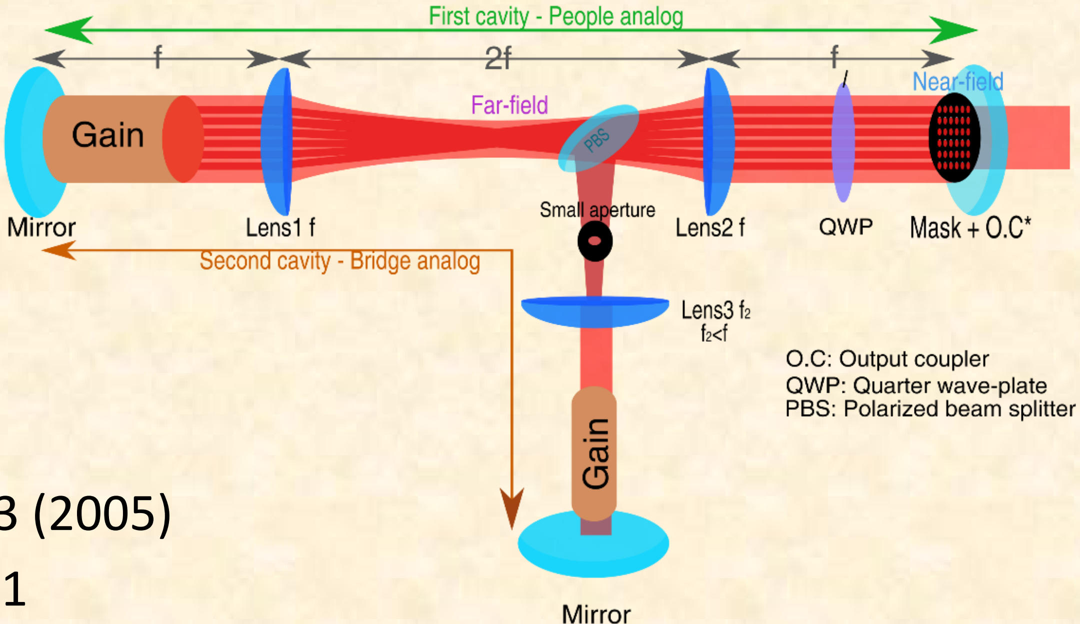
Adding disorder



E. Ronen et. al. unpublished

Kuramoto, Arecchi, Strogatz, McCall, …

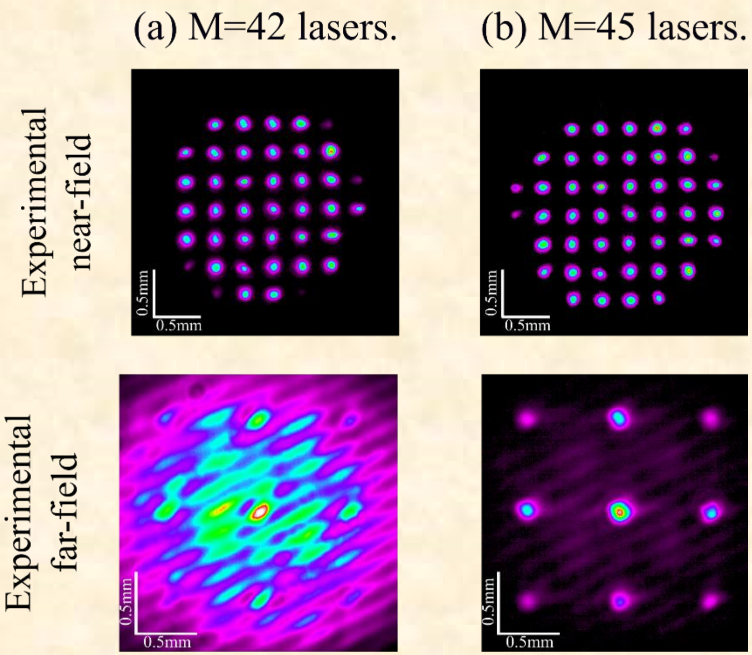
Crowd synchrony

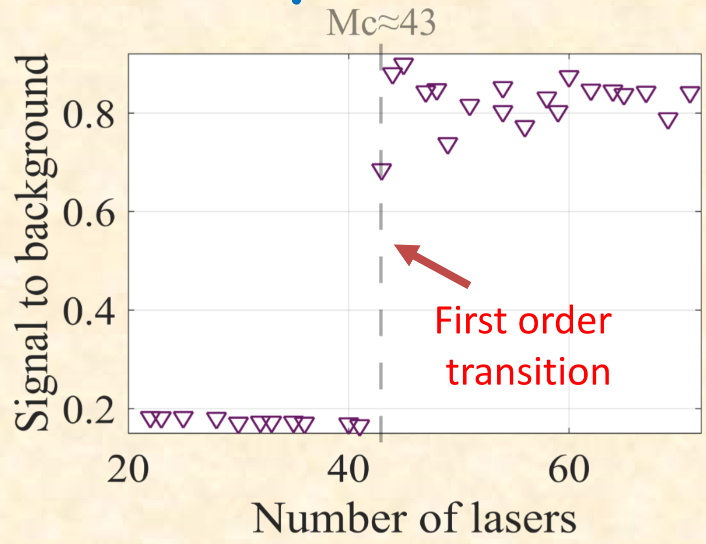


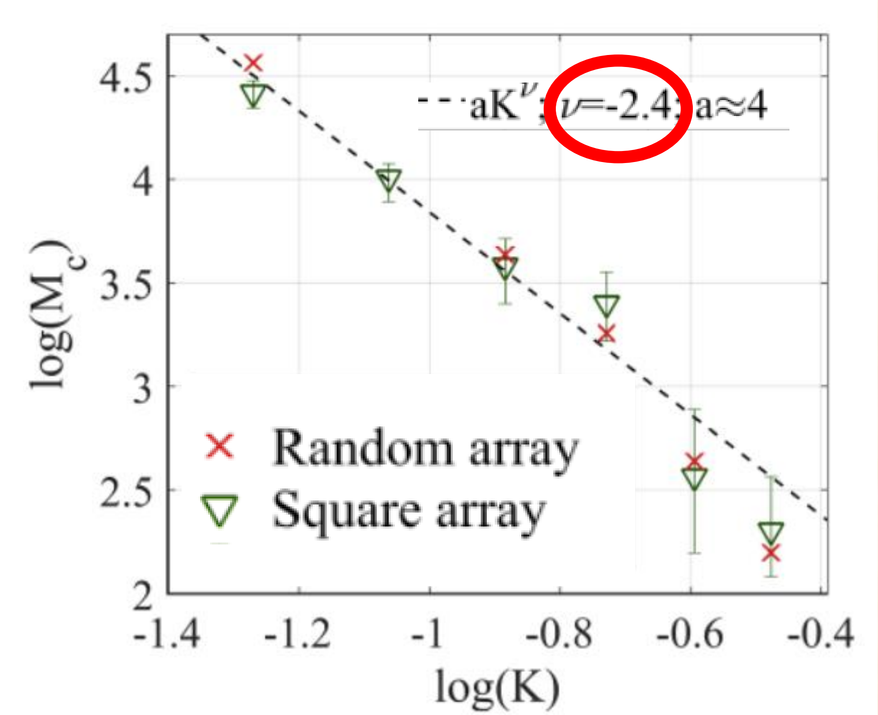
S. Strogatz et. al., Nature 438, 43 (2005)

S. Mahler et. al. arXiv: 2005:0951

Crowd synchrony

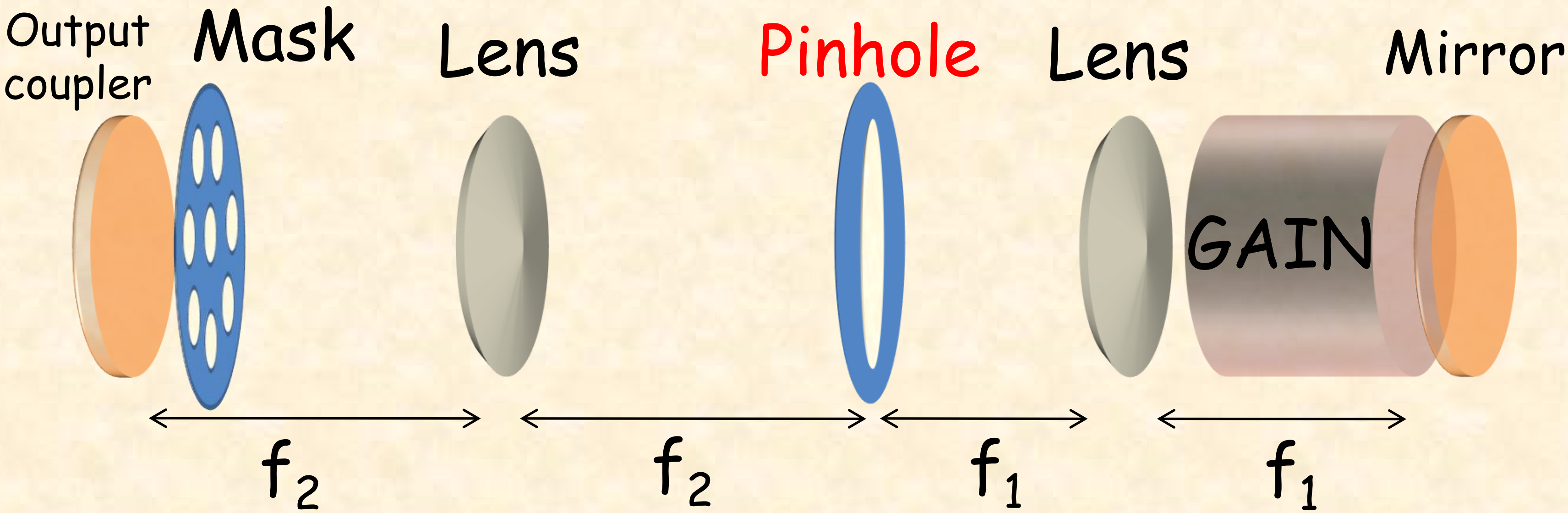




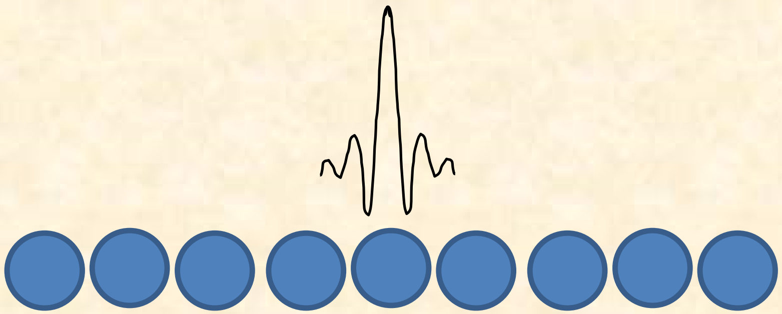


S. Mahler et. al. arXiv: 2005:0951

Long range coupling

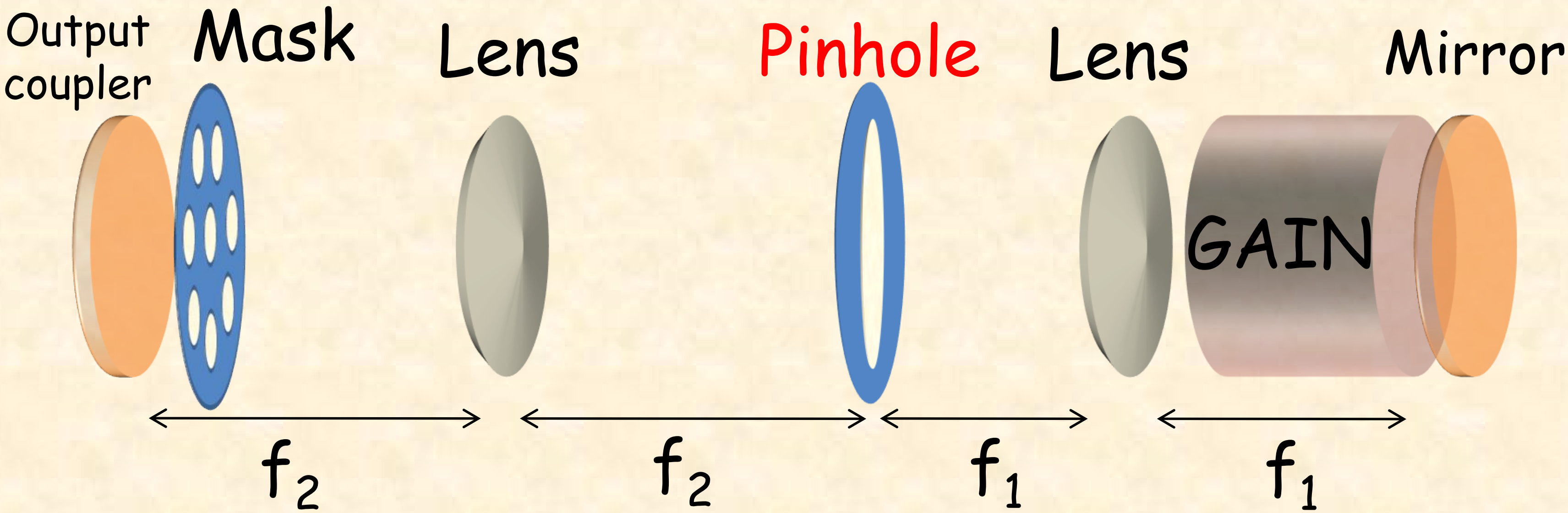


no coupling

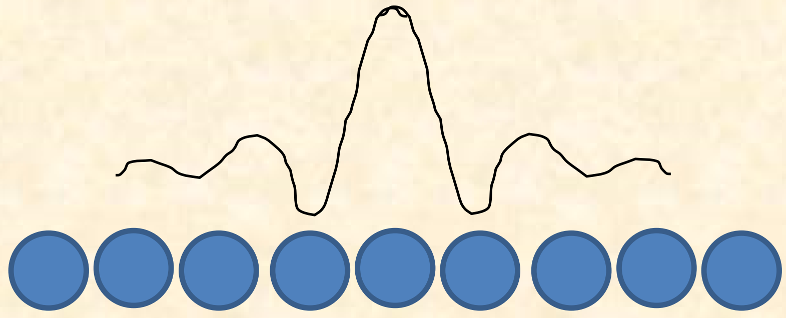


E. Ronen et. al.
unpublished

Long range coupling

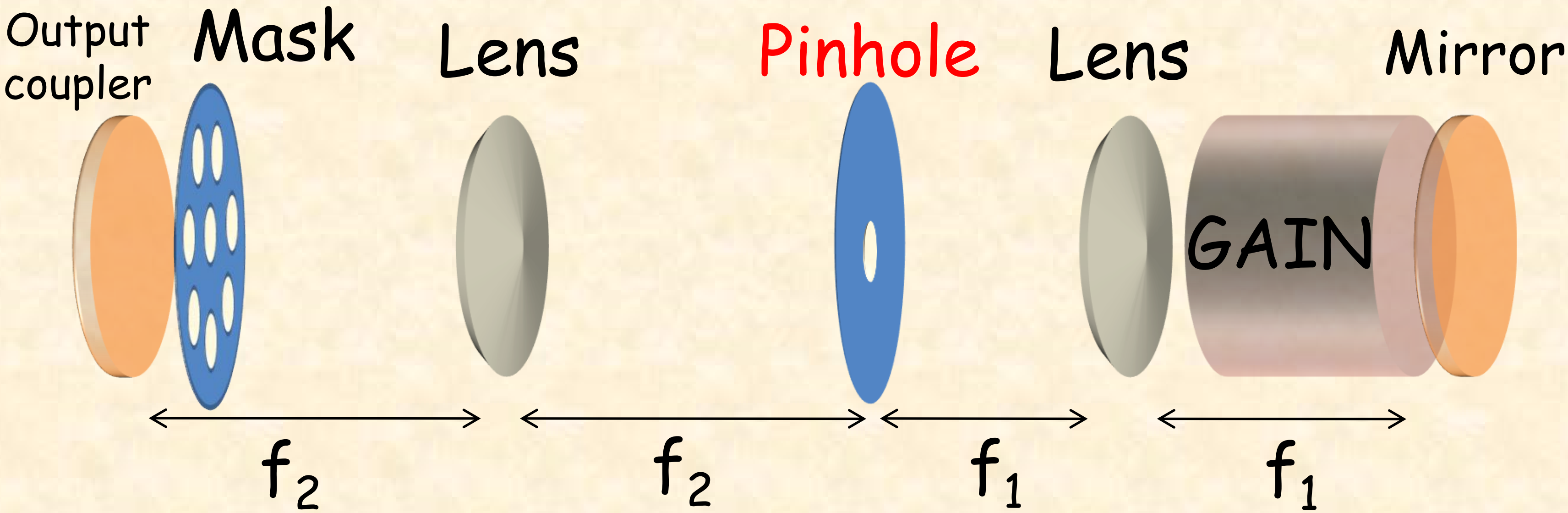


negative coupling

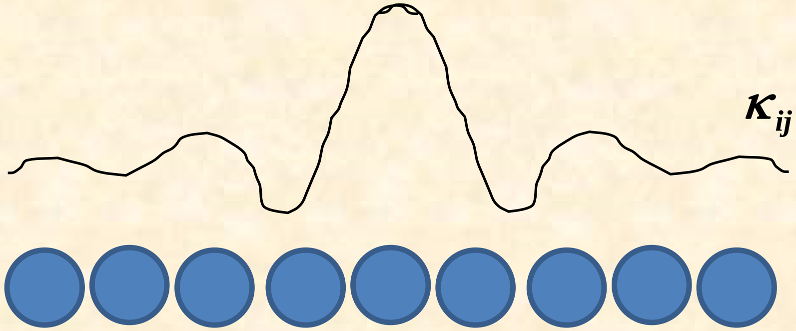


E. Ronen et. al.
unpublished

Long range coupling



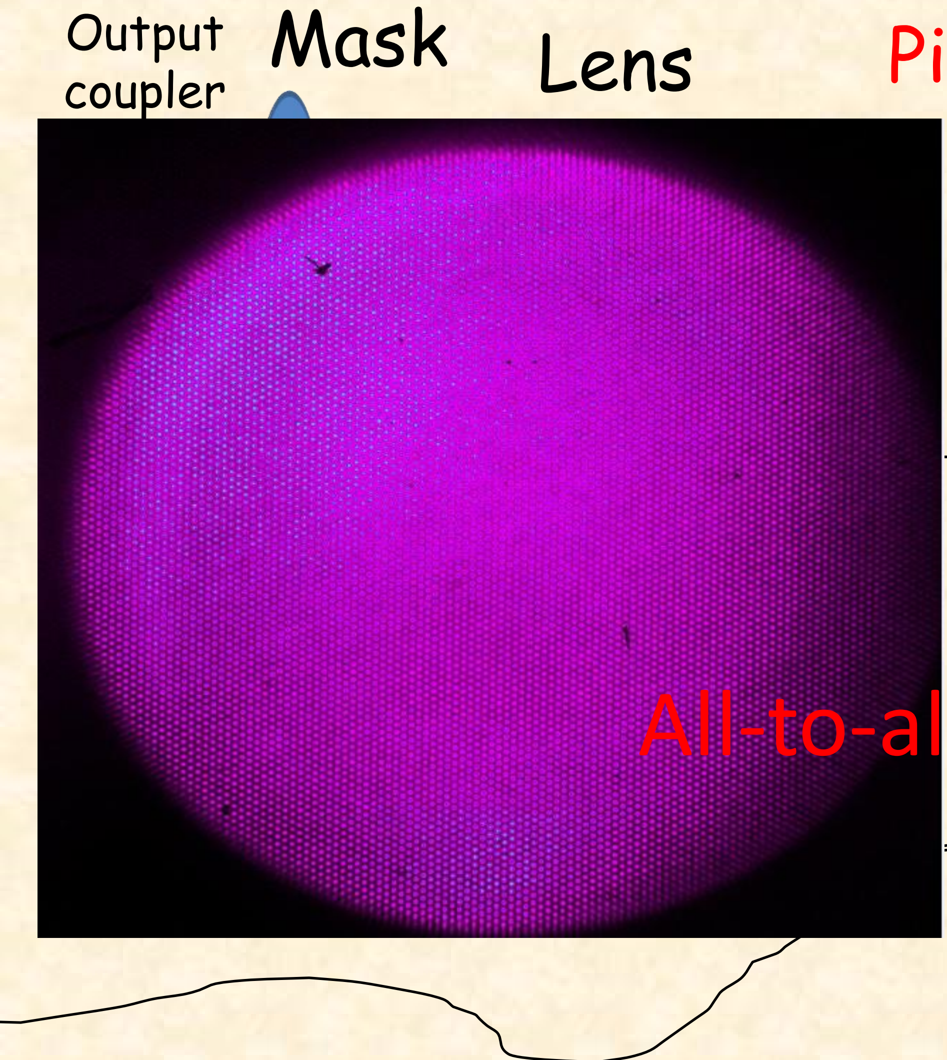
Incommensurate coupling: **spin glass**



$$\kappa_{ij} = \left\langle E_i^{fin} \middle| E_j^{ini} \right\rangle \cong \frac{1}{(i-j)^2}$$

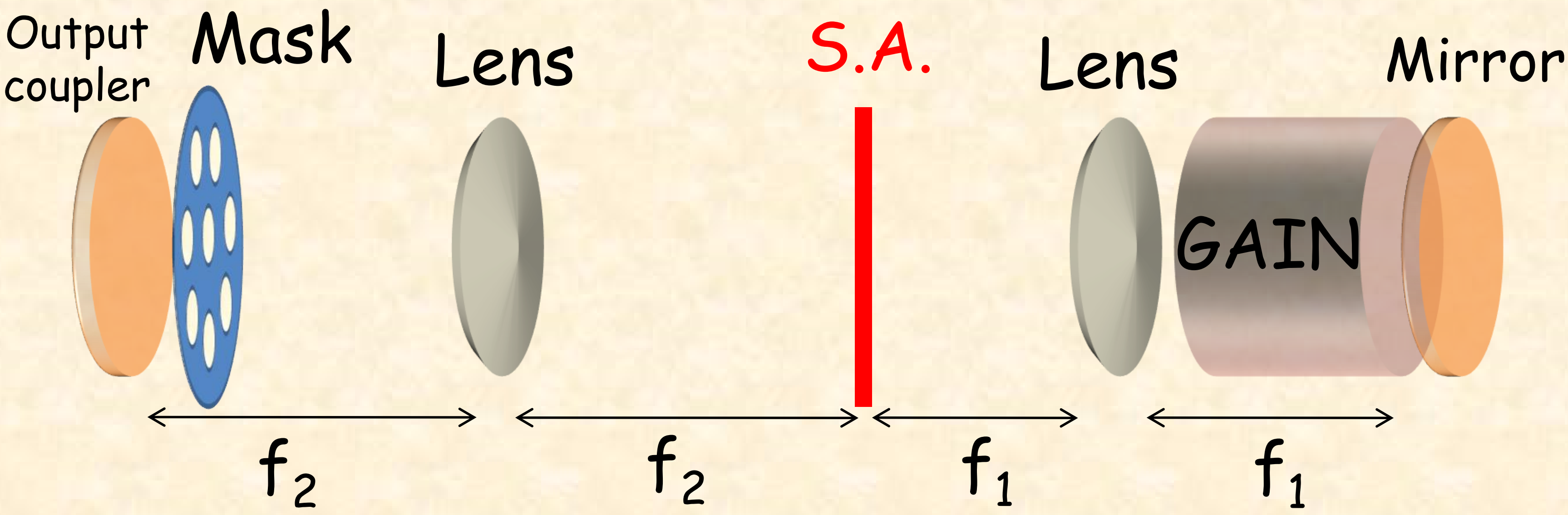
E. Ronen et. al.
unpublished

Long range coupling



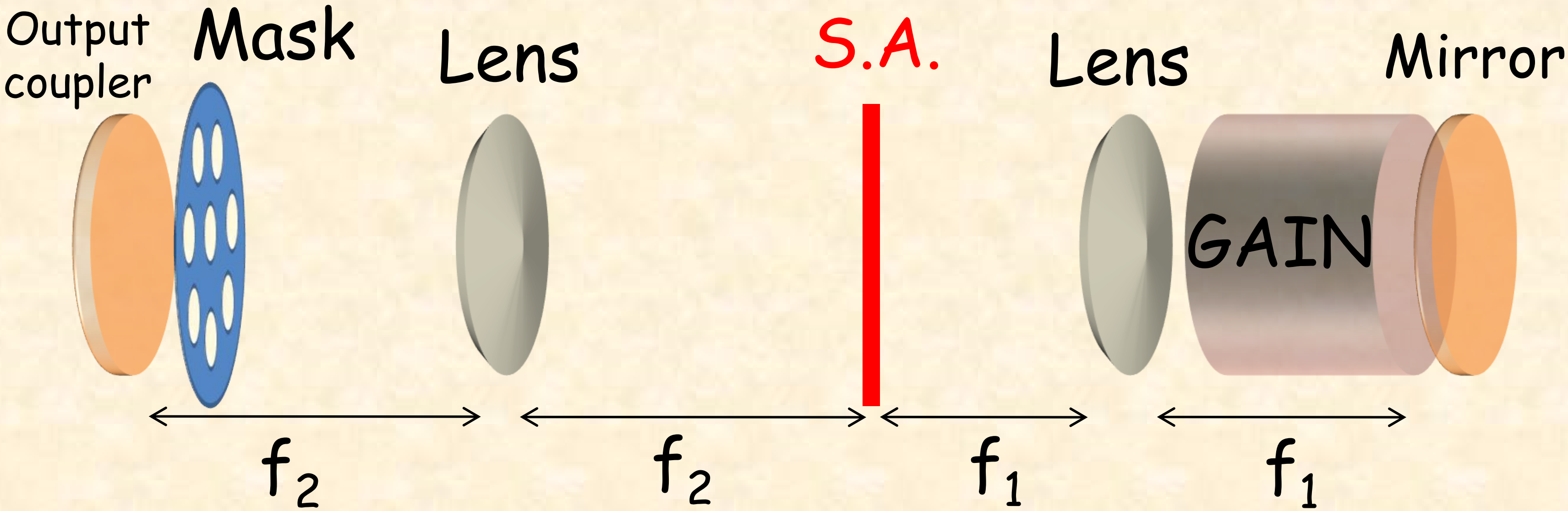
C. Tradonsky et. al.
unpublished

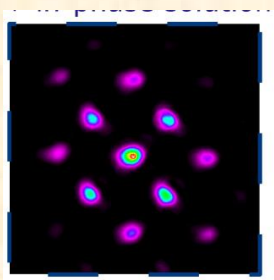
Nonlinear coupling



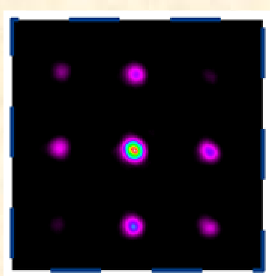
S. Mahler et. al., PRL 124, 133901 (2020).

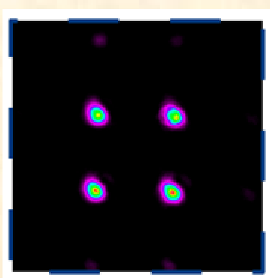
Nonlinear coupling





Linear coupling



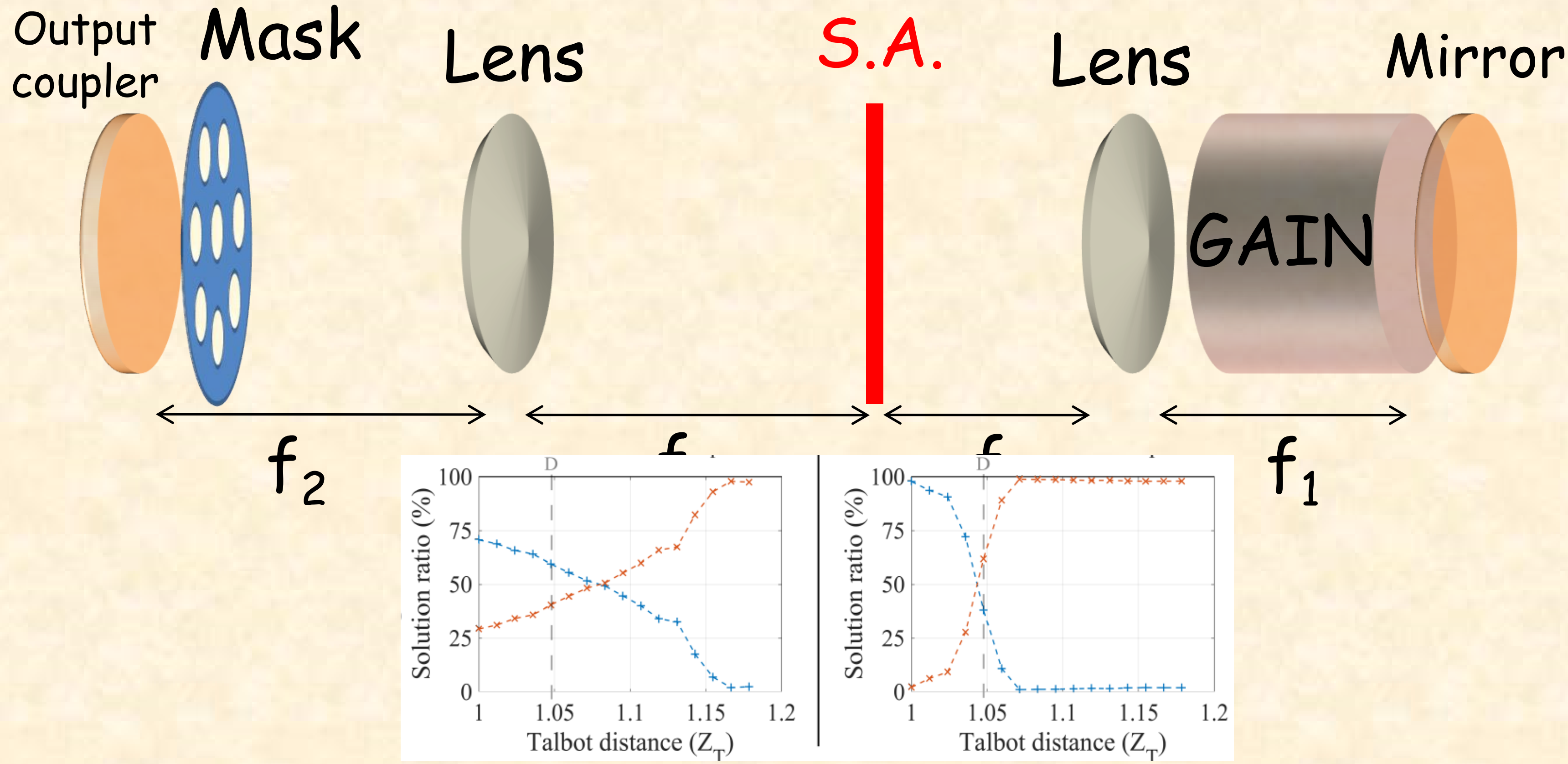


Nonlinear coupling

All longitudinal modes choose same solution

S. Mahler et. al., PRL 124, 133901 (2020).

Nonlinear coupling

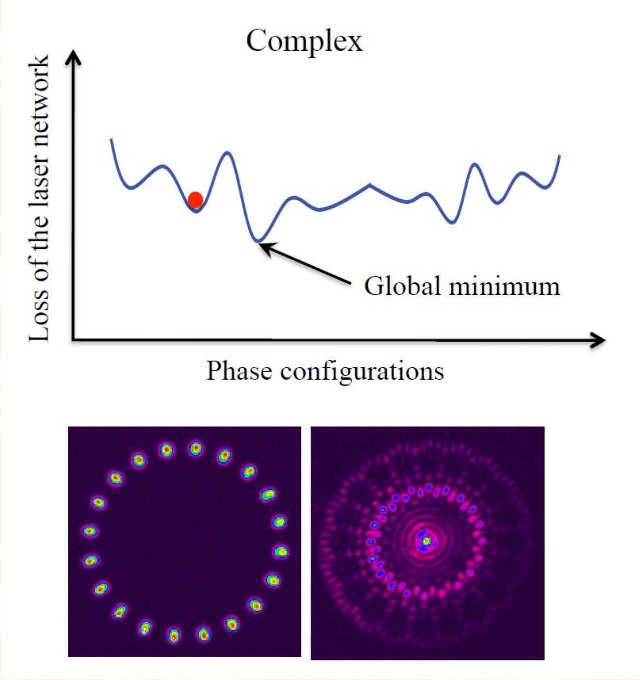


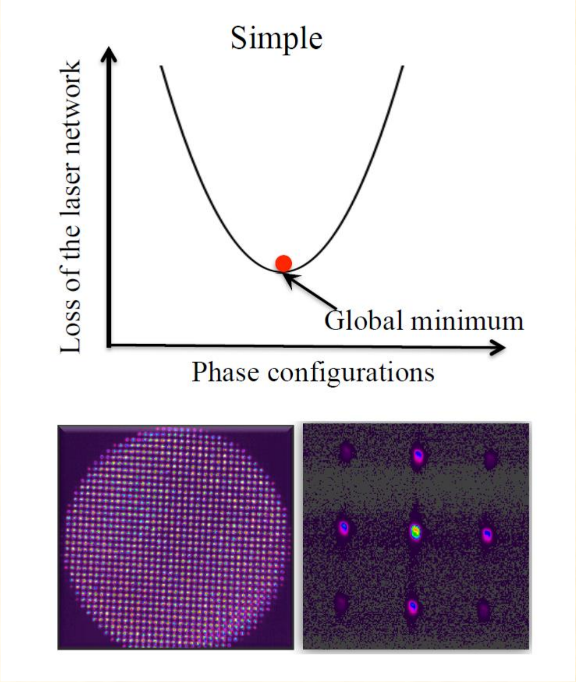
Linear coupling   Nonlinear coupling

×**25** better distinguish near-degenerate solution

S. Mahler et. al., PRL 124, 133901 (2020).

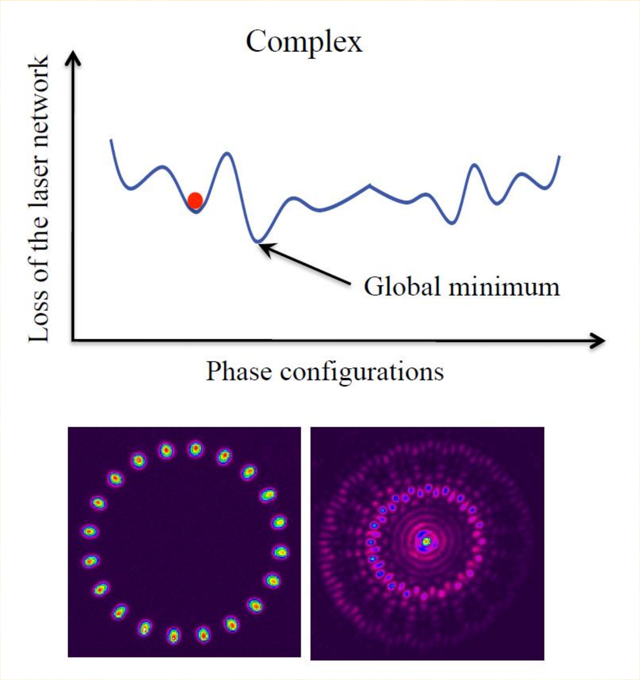
“Hard” problems have complex landscape

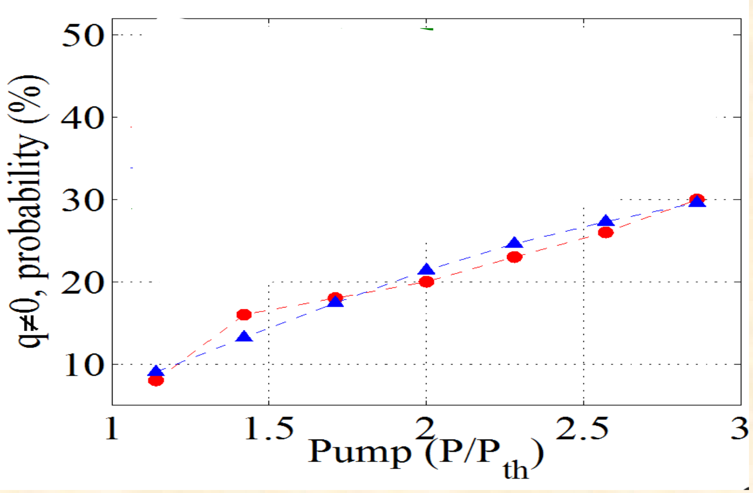


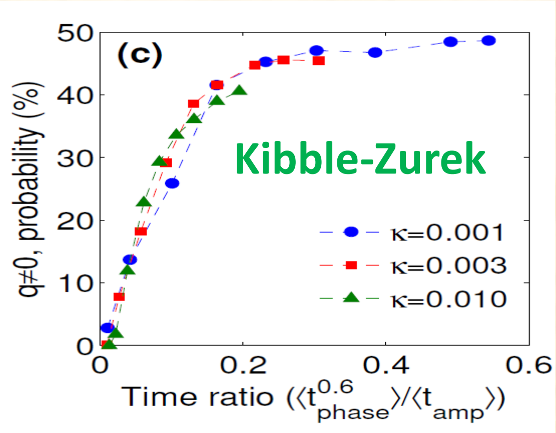


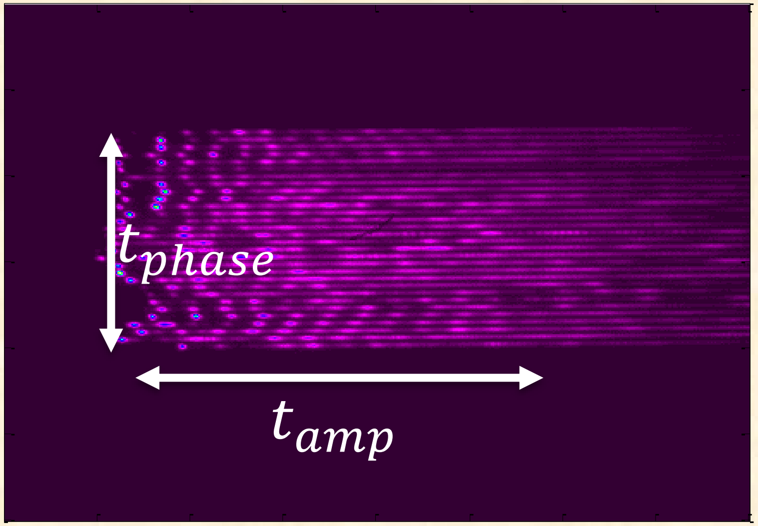
V. Pal et. al., PRL 119, 1013902 (2017)

"Hard" problems have complex landscape



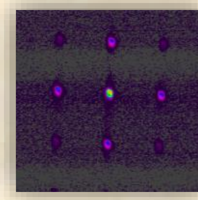


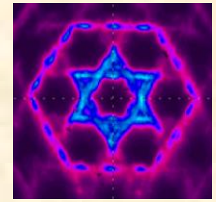


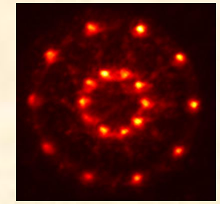


V. Pal et. al., PRL 119, 1013902 (2017)

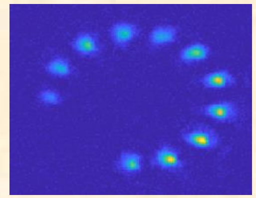
✓ Simulating XY spins

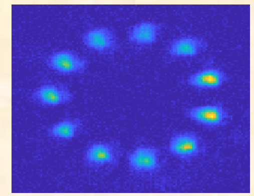




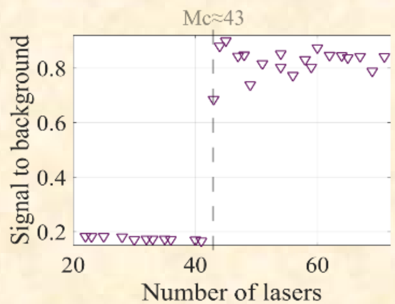


✓ Topology + KZ

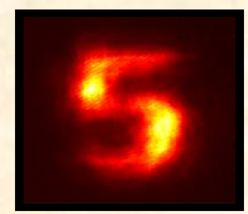




✓ Crowd synchrony



✓ Imaging through tissue



✓ Inverse phase reconstruction