

This talk:

Measuring / characterizing / quantifying
entanglement in quantum simulation?

'Randomized measurement' toolbox

The Credo of Quantum Simulation

Quantum Many-Body System

The system is in a *superposition* state
of all possible configurations ...

Entanglement

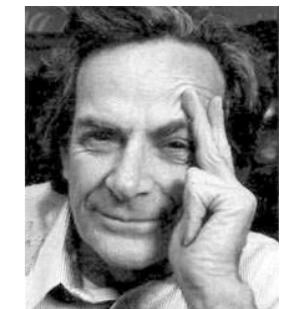


Schrödinger

$$|\Psi\rangle = c_1 \left| \begin{array}{c} \text{A grid of red spheres with up arrows} \end{array} \right\rangle + c_2 \left| \begin{array}{c} \text{A grid of red and blue spheres with mixed up and down arrows} \end{array} \right\rangle + \dots + c_{2^N} \left| \begin{array}{c} \text{A grid of blue spheres with down arrows} \end{array} \right\rangle$$

AND AND AND

Exponentially large Hilbert space



Feynman

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

a thing which is usually described by local differential equations. But the physical world is quantum mechanical, and therefore the proper problem is the simulation of quantum physics—which is what I really want to talk about, but I'll come to that later. So what kind of simulation do I mean?

Entanglement in Quantum Simulation

Quantum Many-Body System

The system is in a *superposition* state
of all possible configurations ...

Entanglement

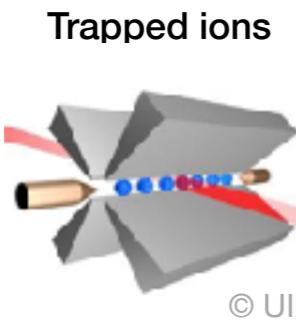


Schrödinger

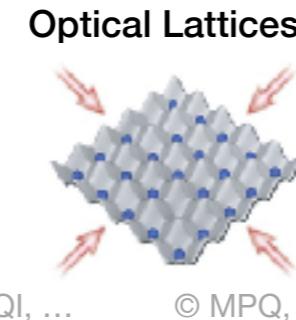
$$|\Psi\rangle = c_1 \left| \text{ (lattice diagram with red up arrows) } \right\rangle + c_2 \left| \text{ (lattice diagram with mixed red and blue up-down arrows) } \right\rangle + \dots + c_{2^N} \left| \text{ (lattice diagram with blue down arrows) } \right\rangle$$

AND AND AND

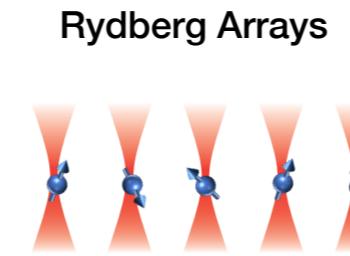
Today we build quantum simulators ...



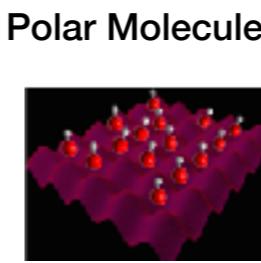
Trapped ions



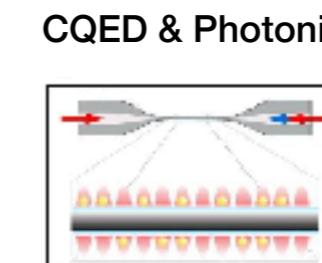
Optical Lattices



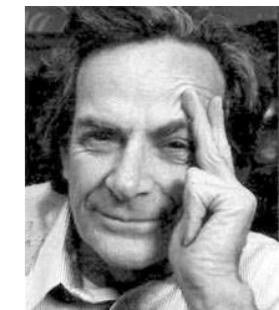
Rydberg Arrays



Polar Molecules



CQED & Photonic



Feynman

How to measure / characterize / quantify entanglement in quantum simulation?

Outline of this talk



A Elben

B Vermersch
→ Grenoble

C Kokail

R. van Bijnen

M. Dalmonte
→ ICTP

Programmable Quantum Simulators [Overview]

Overview: atomic platforms, and opportunities etc.

[see earlier talks: M. Lukin, ...](#)

Randomized Measurement Toolbox

[A. Elben, B. Vermersch et al., 2018-...](#)

Example 1: Measuring Renyi Entanglement Entropies

[Brydges et al., Science 2019](#)

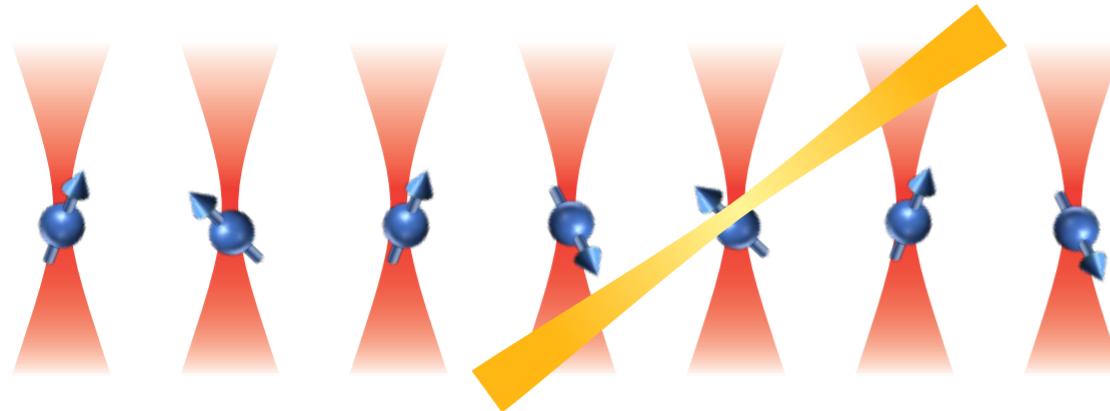
Example 2: Tomography of the Entanglement Hamiltonian

[arXiv:2009.09000](#)

development of theoretical framework, and application to experiment

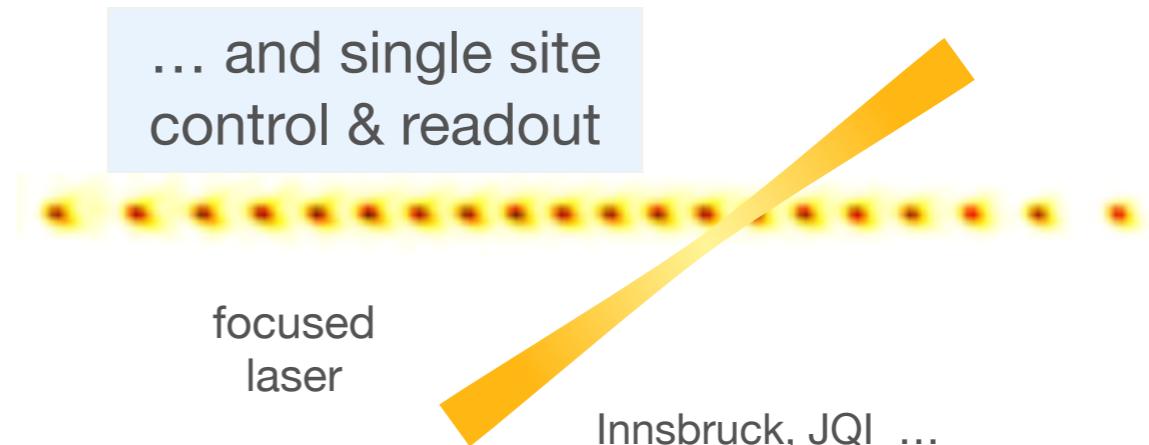
Programmable Analog Quantum Simulators

Rydberg Tweezer Arrays [1D,2D,3D]



Harvard - MIT, Palaiseau, JILA, Caltech, Wisconsin, Sandia, ...

Trapped-ions [1D, 2D]



Engineered Spin Models & Hamiltonians

$$\hat{H} = \sum_i \frac{1}{2} \Omega_i \hat{\sigma}_x^i - \sum_i \Delta_i \hat{n}_i + \sum_{i < j} V_{ij} \hat{n}_i \hat{n}_j$$

$V_{ij} = C_6 / r_{ij}^6$

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + B \sum_i \hat{\sigma}_i^z$$

$J_{ij} \sim \frac{1}{|i-j|^\alpha}$ $\alpha = 0 \dots 3$ long range

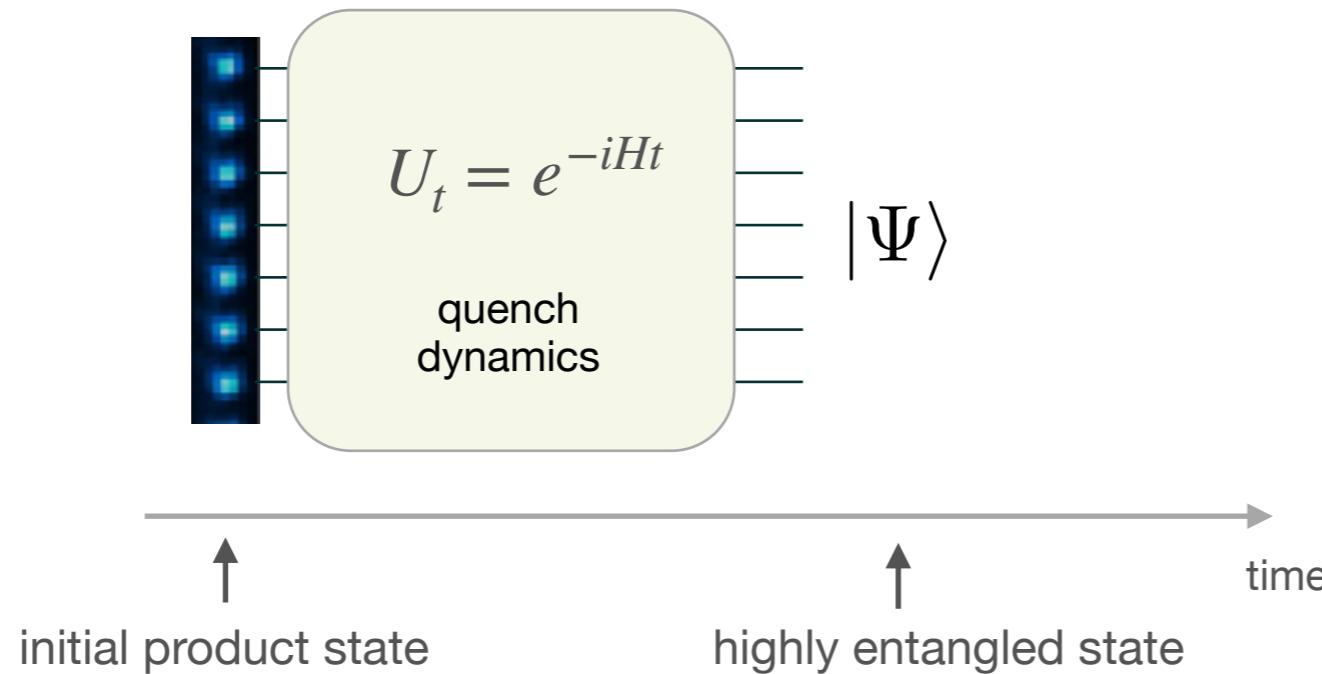
NISQ devices: few tens of atoms, scaling to ~ few hundred (no error correction)

spin-spin interaction via Rydberg-Vander Waals

phonon-mediated spin-spin interaction

Analog Quantum Simulators

Experiments done today ...

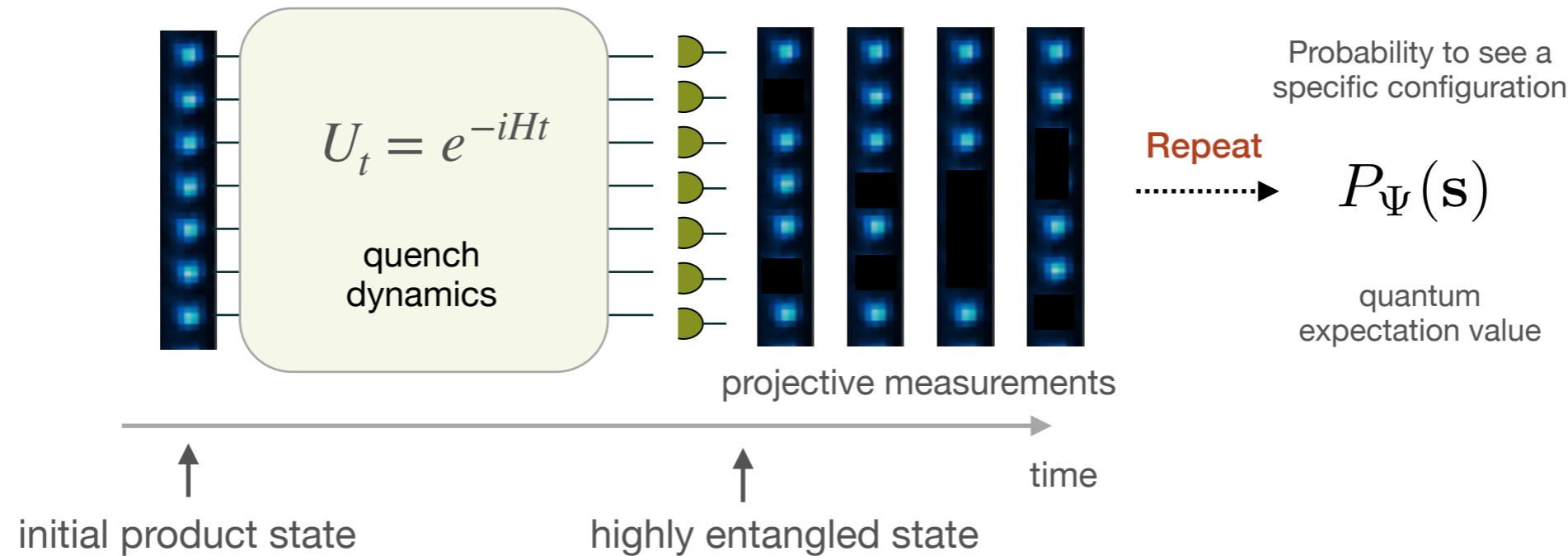


Learn the entanglement properties of quantum state $|\Psi\rangle$

Entanglement as a resource

Analog Quantum Simulators

Experiments done today ...

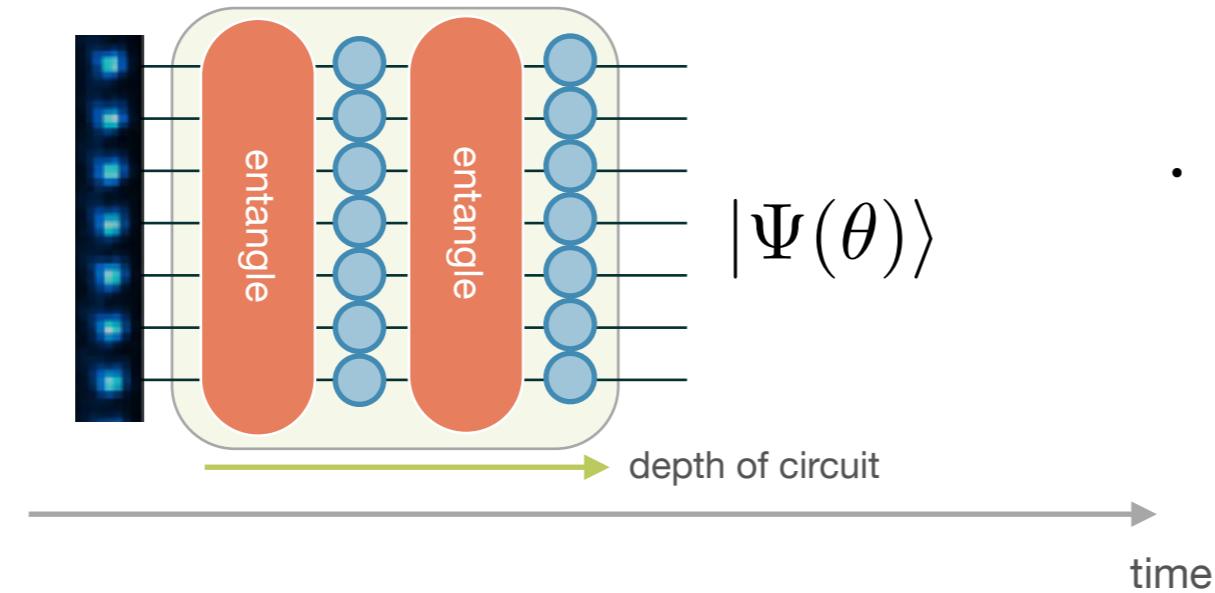


Learn the entanglement properties of quantum state $|\Psi\rangle$

Entanglement as a resource

Programming Quantum Simulators

Complex quantum circuits from available resources

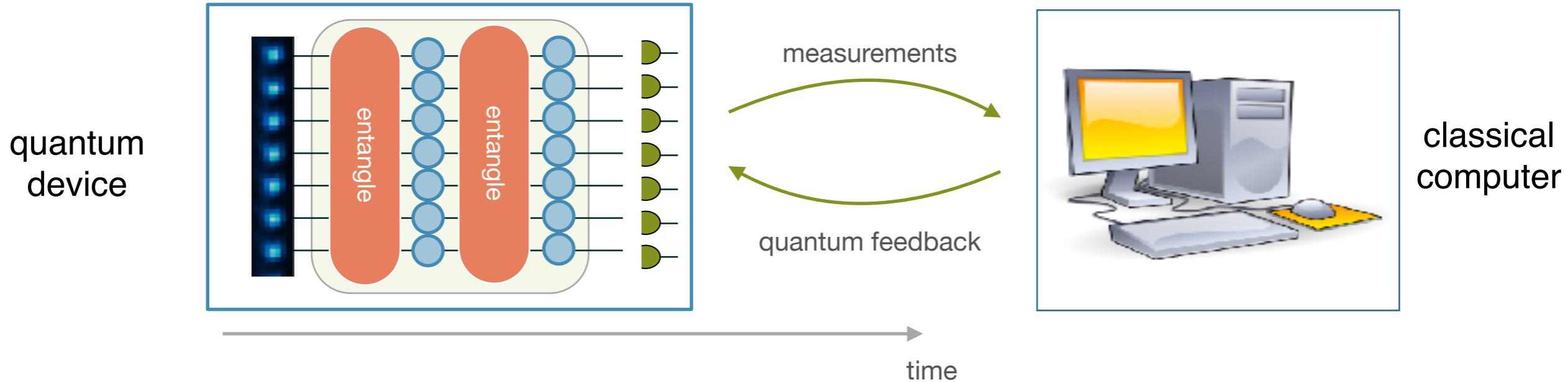


- we can build highly entangled quantum states from available quantum resources parametrized by $\theta_1, \theta_2, \dots$

Entanglement as a resource

Programming Quantum Simulators

Complex quantum circuits from available resources



Variational Classical-Quantum Algorithms

cost function: $C(\theta) = \dots \rightarrow \min$

optimize on classical machine
variational parameters

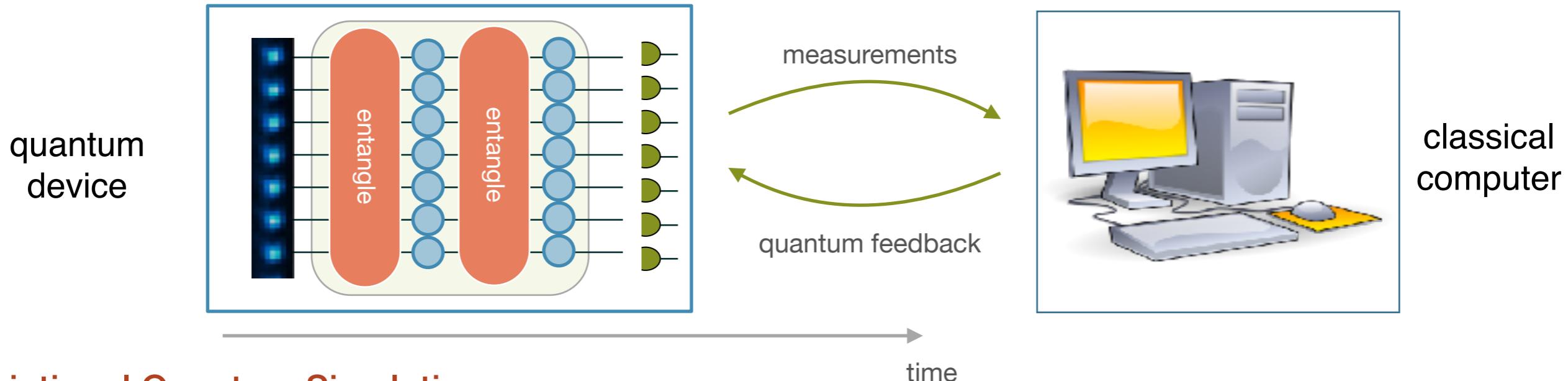
Farhi et al., arXiv:1411.4028

McClean et al. *NJP* (2016)

evaluate on quantum machine
... efficiently

Programming Quantum Simulators

Complex quantum circuits from available resources



Variational Quantum Simulation

target Hamiltonian (e.g. lattice model)

$$\hat{H}_T = \sum_{n\alpha} h_n^\alpha \hat{\sigma}_n^\alpha + \sum_{n\ell\alpha\beta} h_{n\ell}^{\alpha\beta} \hat{\sigma}_n^\alpha \hat{\sigma}_\ell^\beta + \dots$$

cost function with
variational parameters θ

$$C(\theta) = \langle \psi(\theta) | \hat{H}_T | \psi(\theta) \rangle \rightarrow \min$$

Hamiltonian never
physically realized

lowest energy
~ ground state

on quantum computer / chemistry:
JR McClean *et al.*, NJP 2016

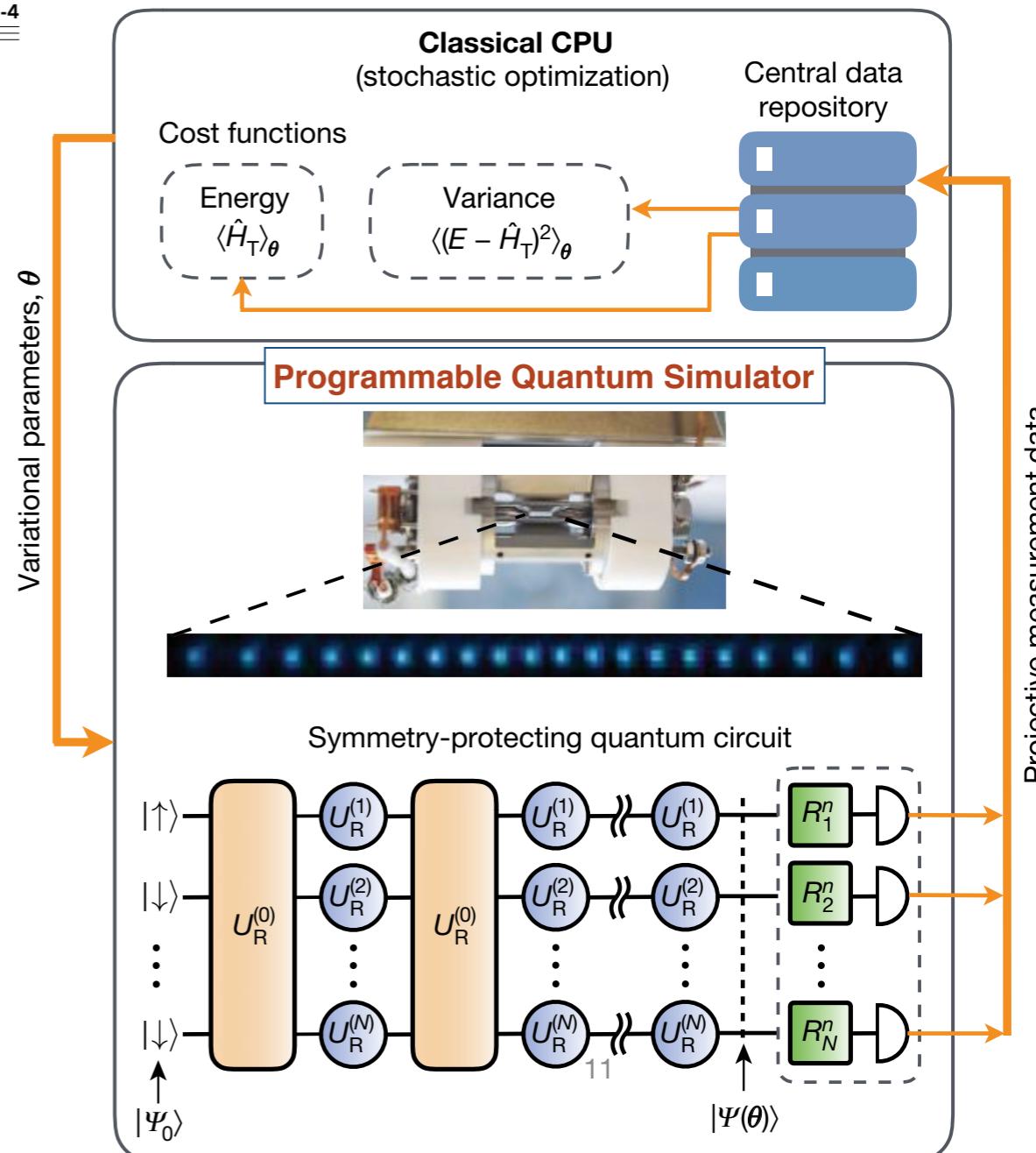
Self-verifying variational quantum simulation of lattice models

C. Kokail^{1,2,3}, C. Maier^{1,2,3}, R. van Bijnen^{1,2,3}, T. Brydges^{1,2}, M. K. Joshi^{1,2}, P. Jurcevic^{1,2}, C. A. Muschik^{1,2}, P. Silvi^{1,2}, R. Blatt^{1,2}, C. F. Roos^{1,2} & P. Zoller^{1,2*}



Rick van Bijnen (th-postdoc), Christine Maier (exp-PhD), Christian Kokail (th-PhD)

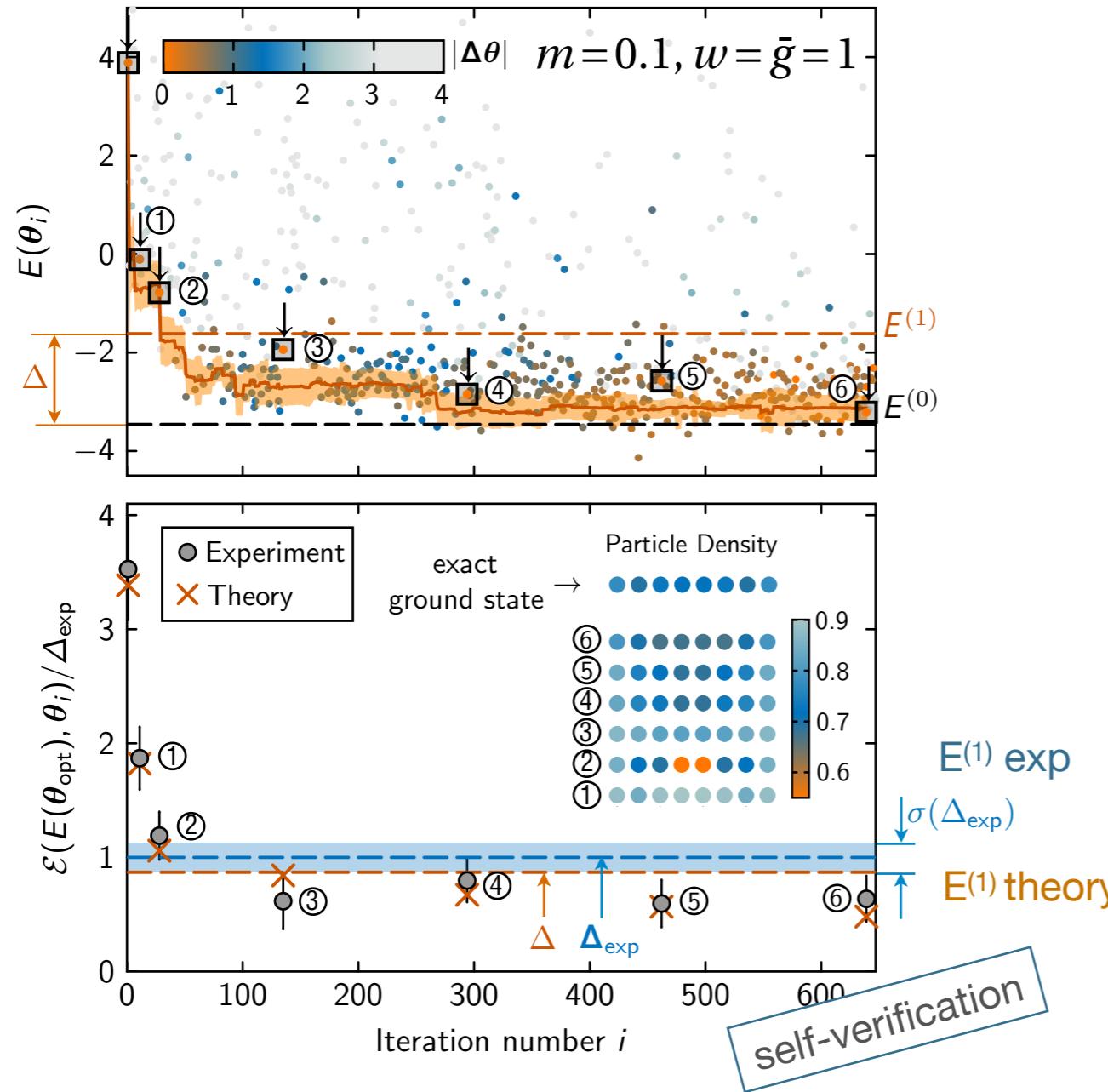
Classical - Quantum Feedback Loop



20 (now: 50) qubits, 10^5 call of PQS, circuit depth 6

Optimization Trajectory for Schwinger Ground State

up to 20 ions, 15 parameters, circuit depth = 6, budget: 10^5 calls to quantum simulator



- optimal energy

$$E_{\boldsymbol{\theta}}^{(0)} = \langle \Psi_{\boldsymbol{\theta}} | \hat{H}_T | \Psi_{\boldsymbol{\theta}} \rangle \rightarrow \min$$

- measure the error bar as energy variance

$$(\Delta E_{\boldsymbol{\theta}}^{(0)})^2 = \langle \Psi_{\boldsymbol{\theta}} | (E_{\boldsymbol{\theta}}^{(0)} - \hat{H}_T)^2 | \Psi_{\boldsymbol{\theta}} \rangle \geq 0$$

$=0$ for eigenstate



A Elben
→ Grenoble

B Vermersch
→ Grenoble

C Kokail

R. van Bijnen M. Dalmonte
→ ICTP

New Many-Body Physics

- Measuring / quantifying entanglement

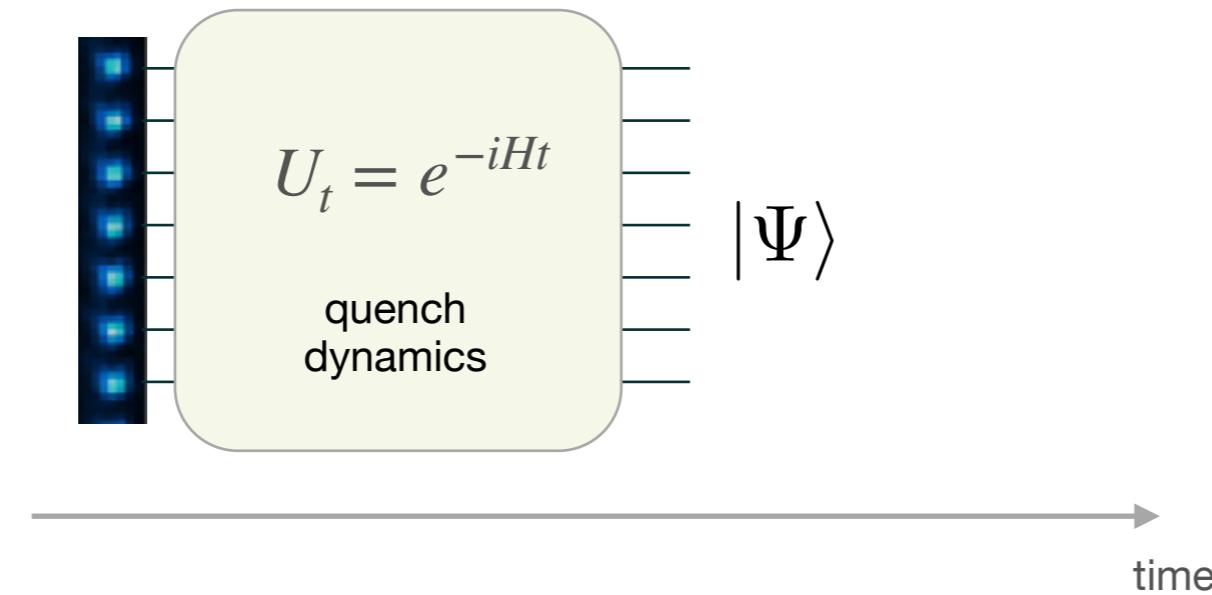
$$|\Psi\rangle = c_1 \left| \begin{array}{c} \text{A 4x4 grid of red spheres with up arrows} \end{array} \right\rangle + c_2 \left| \begin{array}{c} \text{A 4x4 grid with mixed red and blue spheres, some up, some down} \end{array} \right\rangle + \dots + c_{2^N} \left| \begin{array}{c} \text{A 4x4 grid of blue spheres with down arrows} \end{array} \right\rangle$$

AND AND AND

... enabled by *programmable q-simulators & classical postprocessing*

Programmable Analog Quantum Simulator

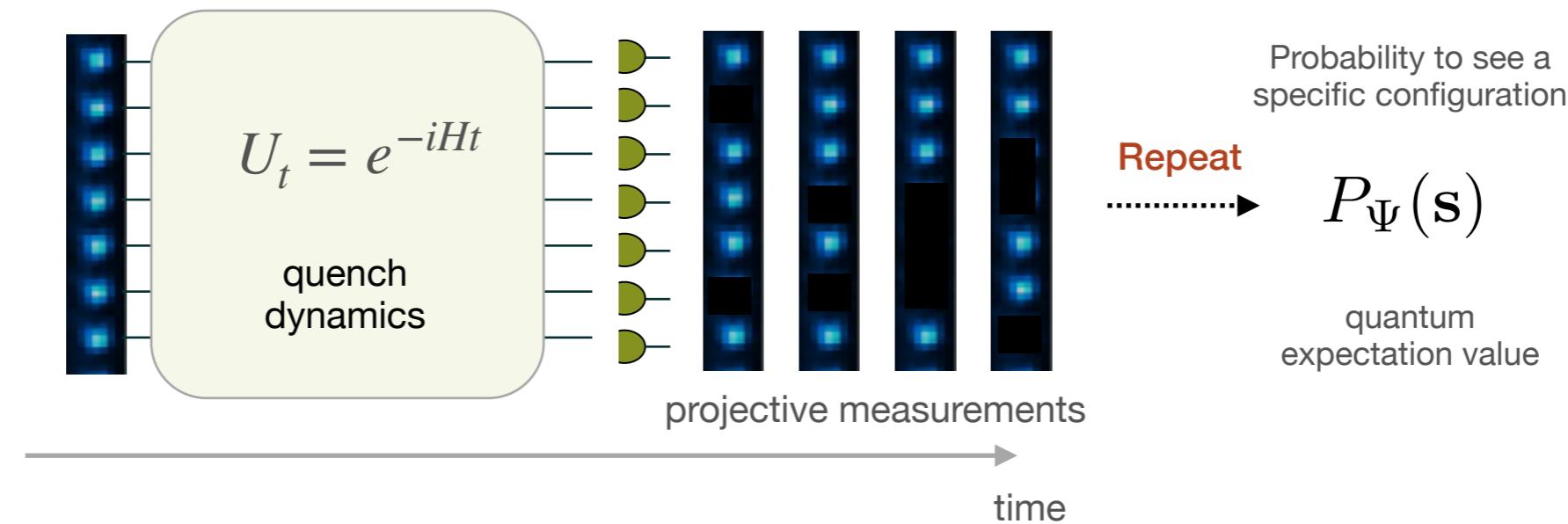
Experiments done today ...



Learn the entanglement properties of quantum state $|\Psi\rangle$?

Programmable Analog Quantum Simulator

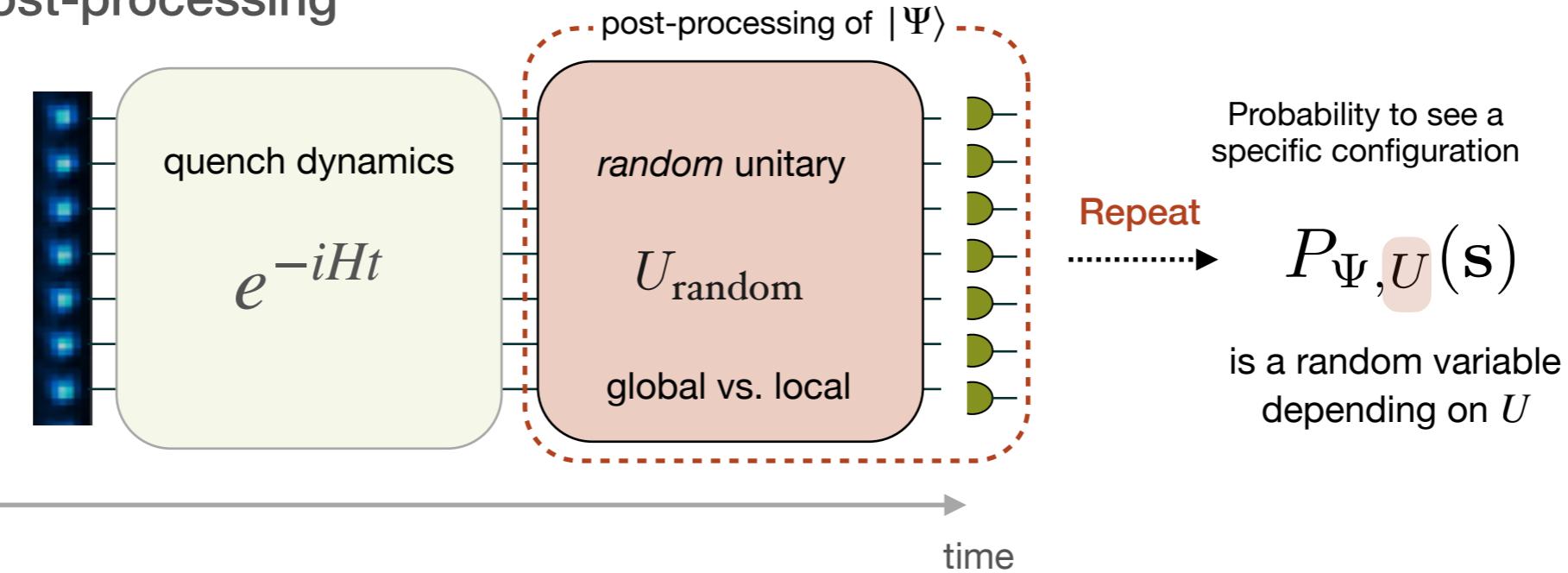
Experiments done today ...



Learn the entanglement properties of quantum state $|\Psi\rangle$?

Randomized Measurements

Measurement post-processing



(Cross-) Correlation of probabilities

'Noise' or ensemble average (e.g. CUE)

$$\overline{P_{\Psi,U}(\mathbf{s}_1)P_{\Psi,U}(\mathbf{s}_2)}$$

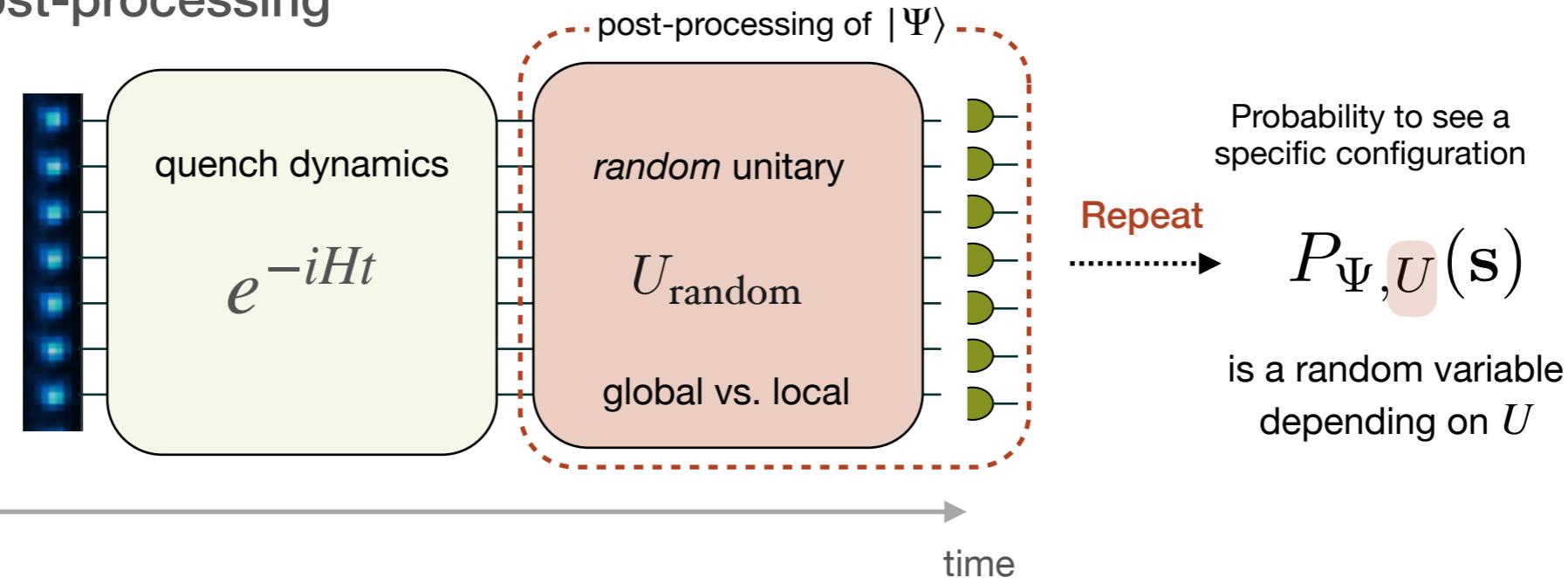
experiment 'day 1, lab 1'

experiment 'day 2 lab 2'

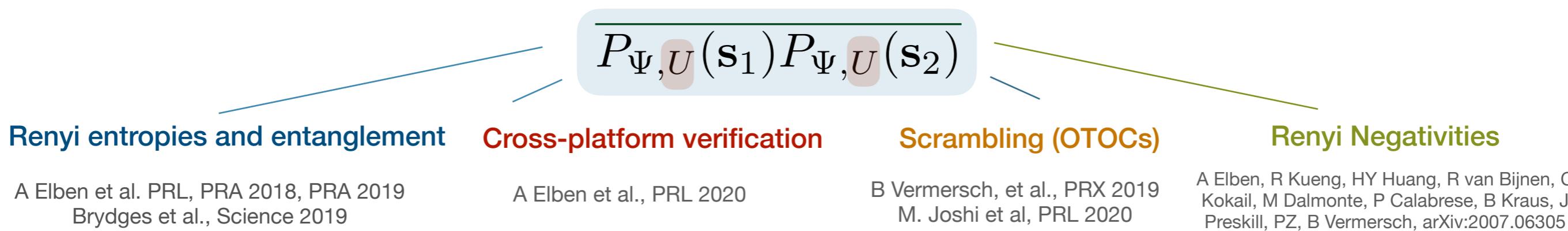
... hybrid classical-quantum protocols

Randomized Measurements

Measurement post-processing



(Cross-) Correlation of probabilities



Entanglement Properties of Quantum State

Quantum system in a pure state

$$|\Psi\rangle = \sum_{\mu_A, \mu_B} c_{\mu_A, \mu_B} |\mu_A\rangle \otimes |\mu_B\rangle$$

Schmidt decomposition

$$|\Psi\rangle = \sum_{\alpha=1}^{\chi_A} \lambda_\alpha |\Phi_\alpha^A\rangle \otimes |\Phi_\alpha^B\rangle$$

Schmidt values

$\chi_A = 1$ product state
 $\chi_A > 1$ entangled state

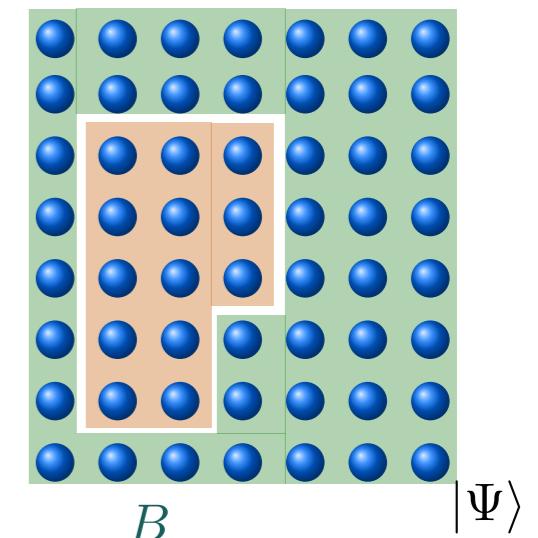
Reduced density matrix

$$\rho_A = \text{Tr}_B [|\Psi\rangle\langle\Psi|] \equiv \sum_{\alpha=1}^{\chi_A} \lambda_\alpha^2 |\Phi_\alpha^A\rangle\langle\Phi_\alpha^A|$$

density matrix of rank χ_A

L. Amico, R. Fazio, A. Osterloh, and V. Vedral, RMP (2008) V. Eisler & I. Peschel, J Phys A (2017).
B. Zeng et al., Quantum information meets quantum matter (Springer, 2019).

bipartition $A:B$



... what should we measure?

Entanglement Properties of Quantum State

Quantum system in a pure state

$$|\Psi\rangle = \sum_{\mu_A, \mu_B} c_{\mu_A, \mu_B} |\mu_A\rangle \otimes |\mu_B\rangle$$

Schmidt decomposition

$$|\Psi\rangle = \sum_{\alpha=1}^{\chi_A} \lambda_\alpha |\Phi_\alpha^A\rangle \otimes |\Phi_\alpha^B\rangle$$

$\chi_A = 1$ product state

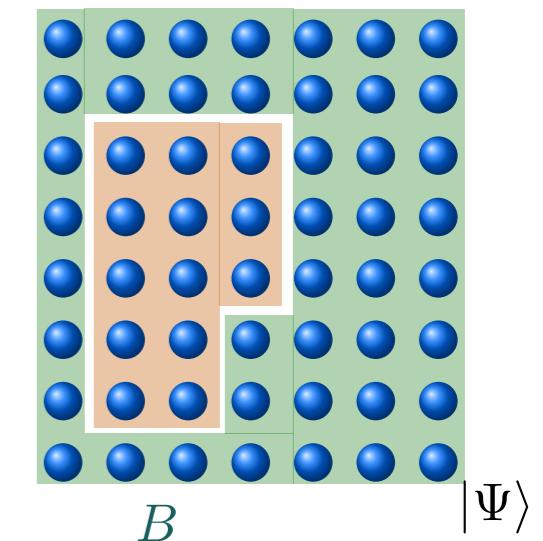
$\chi_A > 1$ entangled state

Interpretation as a Thermal State

$$\rho_A = e^{-\tilde{H}_A} = \sum_{\alpha=1}^{\chi_A} e^{-\xi_\alpha} |\Phi_\alpha^A\rangle \langle \Phi_\alpha^A| \lambda_\alpha^2$$

with the Entanglement Hamiltonian \tilde{H}_A

bipartition $A:B$



... what should we measure?

Entanglement Properties of Quantum State

Entanglement entropies

$$S_A^{(n)} = -\frac{1}{n-1} \log \text{Tr}_A \rho_A^n$$

$$S_A^{\text{VN}} = -\text{Tr}_A (\rho_A \log \rho_A)$$

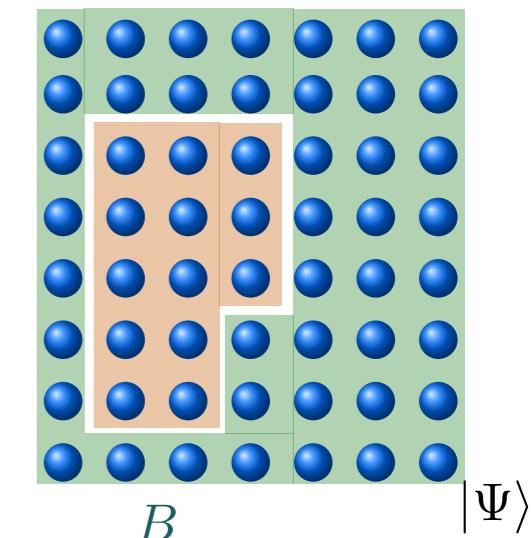
Renyi for $n=2$: purity $\text{Tr}_A \rho_A^2$

Von Neumann

purity of ρ_A

$S_A = 0$ product vs. $S_A > 0$ entangled state, or $\text{Tr}_A \rho_A^2 < 1$

bipartition $A:B$



Entanglement spectrum

$$\rho_A = e^{-\tilde{H}_A} = \sum_{\alpha=1}^{\chi_A} e^{-\xi_\alpha} |\Phi_\alpha^A\rangle\langle\Phi_\alpha^A|$$

tomography of EH

entanglement spectrum

Rem.: tomography $\sim \chi_A 2^{N_A}$

... what & how measure?

... with what resources?

Entanglement Properties of Quantum State

increasing complexity / information ↓

Entanglement entropies

$$S_A^{(n)} = -\frac{1}{n-1} \log \text{Tr}_A \rho_A^n$$

Renyi for $n=2$: purity $\text{Tr}_A \rho_A^2$

$$S_A^{\text{VN}} = -\text{Tr}_A (\rho_A \log \rho_A)$$

Von Neumann

purity of ρ_A

$S_A = 0$ product vs. $S_A > 0$ entangled state, or $\text{Tr}_A \rho_A^2 < 1$

Entanglement spectrum and Hamiltonian

$$\rho_A = e^{-\tilde{H}_A} = \sum_{\alpha=1}^{\chi_A} e^{-\xi_\alpha} |\Phi_\alpha^A\rangle\langle\Phi_\alpha^A|$$

tomography of EH

Rem.: tomography $\sim \chi_A 2^{N_A}$

entanglement spectrum

Why measure entanglement spectrum?

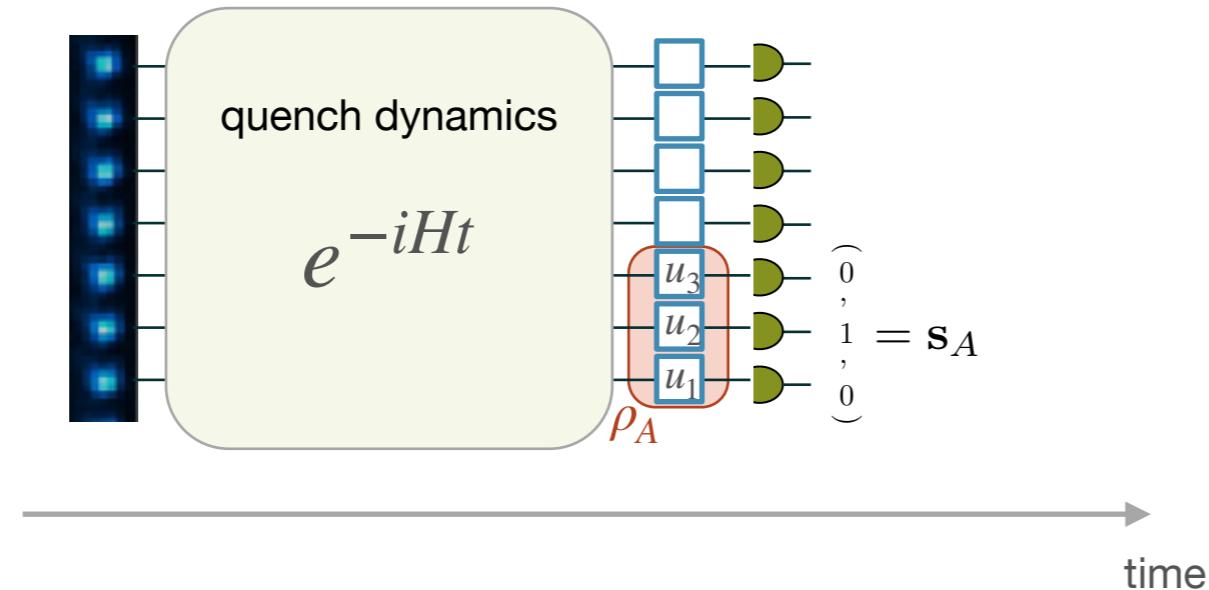
- Entanglement measures
- detection of topological phases
- detection of quantum phase transitions
- extremely hard to obtain numerically

N. Regnault, arXiv:1510.07670 (Les Houches Lectures)

... protocol to measure?
... with what resources?

Randomized Measurements for second order Renyi entropy

Measurement post-processing



$$P_U(\mathbf{s}_A) = \text{Tr} \left[U_A \rho_A U_A^\dagger |\mathbf{s}_A\rangle \langle \mathbf{s}_A| \right]$$
$$U_A = \bigotimes u_i \quad u_i \in \text{CUE}(d)$$

Random single spin rotations are sufficient

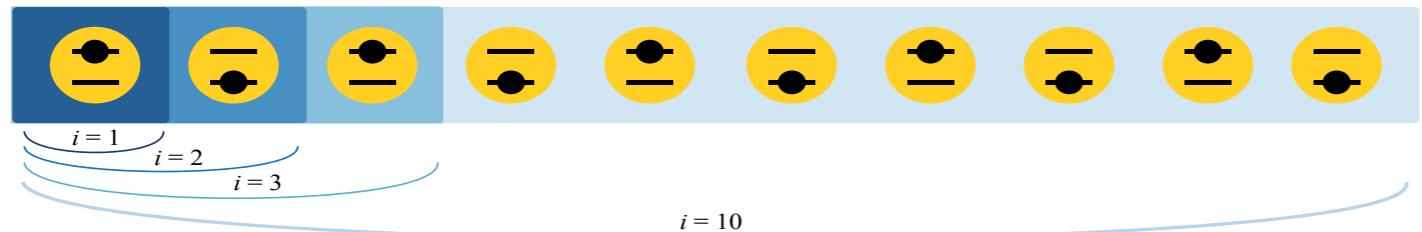
purity $\text{Tr}_A \rho_A^2 = \overline{X_U}$

$$X_U = 2^{N_A} \sum_{s_A, s'_A} (-2)^{-D[s_A, s'_A]} \underbrace{P_U(s_A) P_U(s'_A)}_{\substack{\text{Hamming distance} \\ \text{Cross correlation} \\ \text{between configurations}}}$$

↑
random variable

A. Elben et al. PRA (2019). T Brydges et al., Science (2019)

Quench dynamics with long-range XY-Model

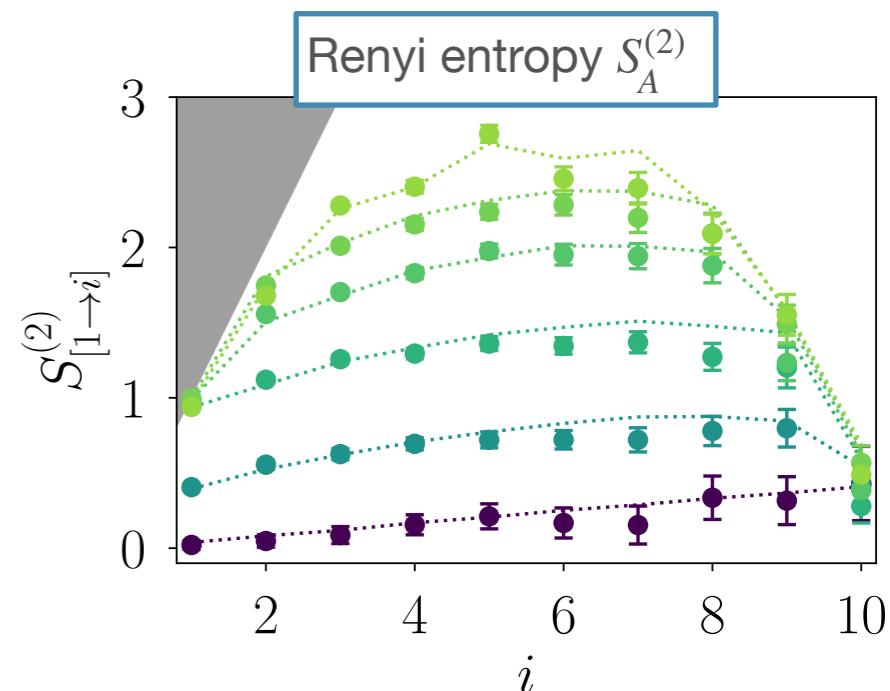
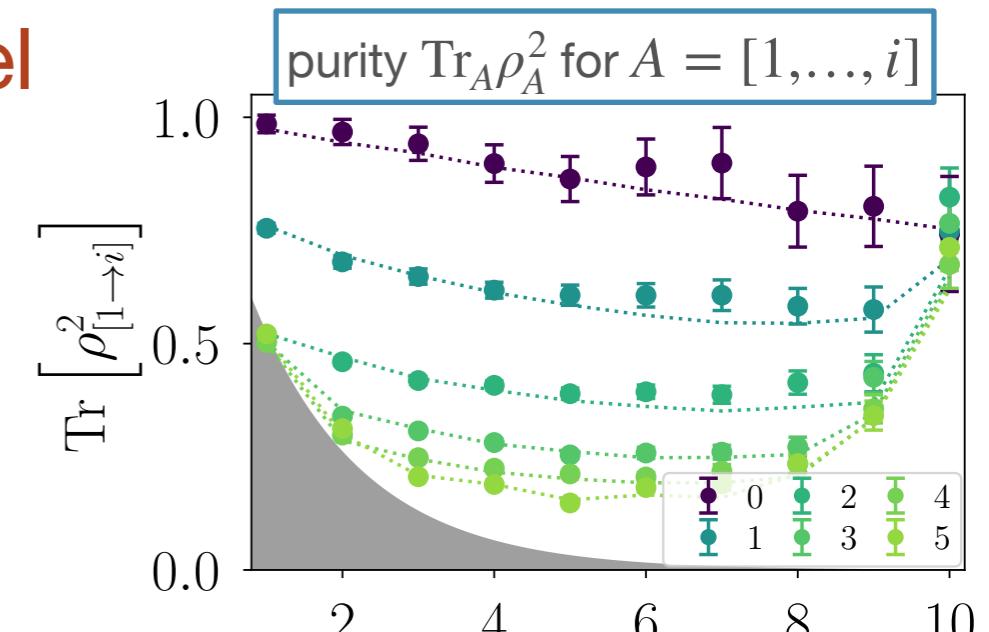


10 - 20 ion quantum simulator:

$$H = \sum_{i < j} (J_{ij}\sigma_i^+ \sigma_j^- + \text{h.c.}) + B \sum_i \sigma_i^z$$

Global quench with $B \gg \max\{J_{ij}\}$

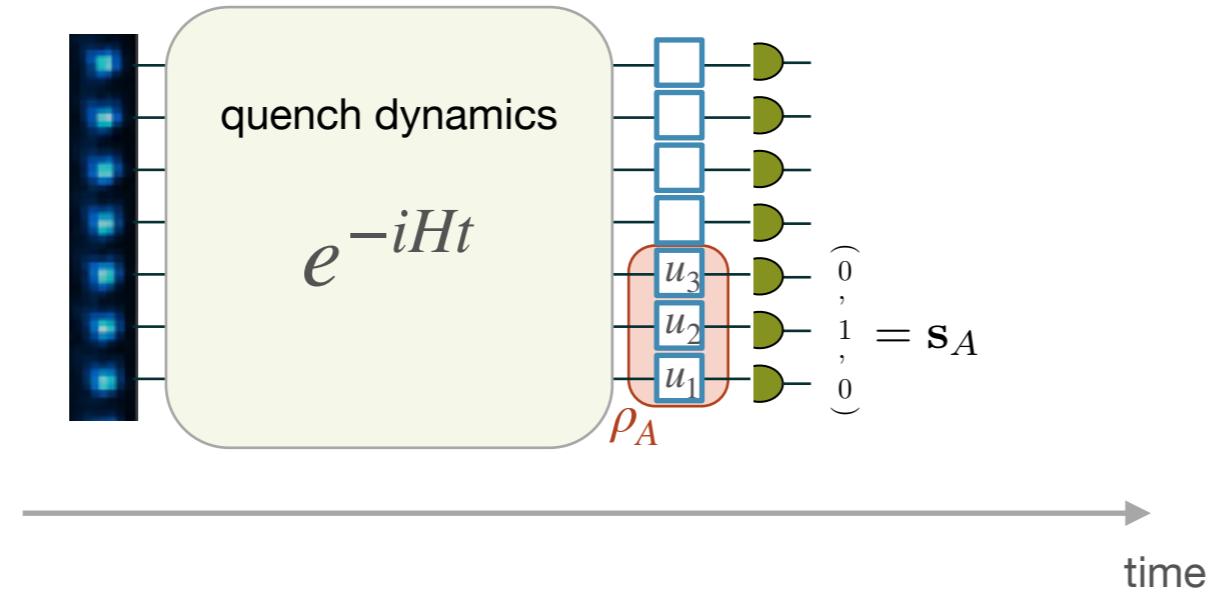
$$|\psi(t)\rangle = e^{-iHt} |\psi_0\rangle \text{ with } |\psi_0\rangle = |\uparrow\downarrow\rangle^{\otimes \frac{N}{2}}$$



T Brydges, A Elben et al., Science 2019

Randomized Measurements

Measurement post-processing



$$P_U(\mathbf{s}_A) = \text{Tr} \left[U_A \rho_A U_A^\dagger |\mathbf{s}_A\rangle\langle\mathbf{s}_A| \right]$$
$$U_A = \otimes u_i \quad u_i \in \text{CUE}(d)$$

Randomized Tomography

A. Elben et al., PRA (2019) (see also Eisert et al.)

Predicting Many Properties of a Quantum System from Very Few Measurements

HY Huang, R Kueng, J Preskill, Nat. Phys. (2020)

Tomography review: A Acharya et al., J Phys A (2019).

$$\rho_A^{RT} = \sum_{\mathbf{s}_A, \mathbf{s}'_A} \sum_{U_A} P_U(\mathbf{s}_A) (-2)^{-D[\mathbf{s}_A, \mathbf{s}'_A]} U_A |\mathbf{s}'_A\rangle\langle\mathbf{s}'_A| U_A^\dagger$$

How many random unitaries N_U and measurements N_M ?

Scaling with subsystem size $|A|$



Entanglement Hamiltonian Tomography

C Kokail R. van Bijnen A. Elben B. Vermersch
→ Grenoble

$$\rho_A = e^{-\tilde{H}_A} = \sum_{\alpha=1}^{\chi_A} e^{-\xi_\alpha} |\Phi_\alpha^A\rangle\langle\Phi_\alpha^A|$$

entanglement Hamiltonian entanglement spectrum

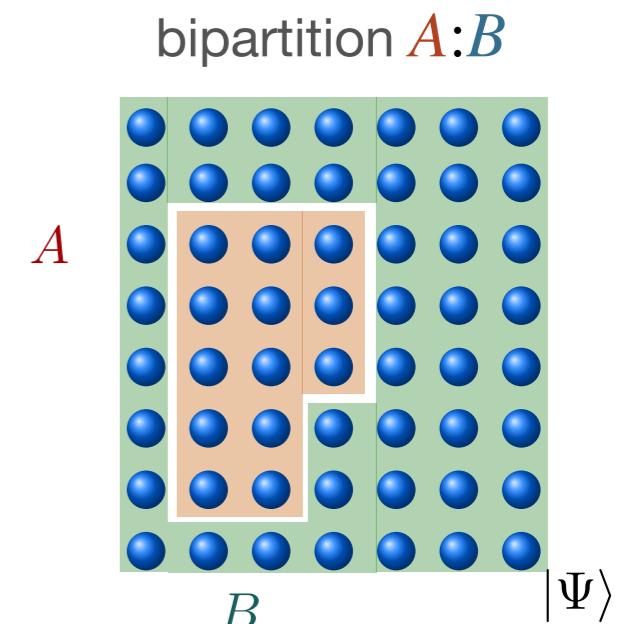
Remarks:

- tomography exponentially expensive with size of A , $\sim \chi_A 2^{N_A}$
 - efficient if we know something about the quantum state (MPS, low rank, ...)
- review: A Acharya et al. J Phys A (2019).

Challenge: *efficient* tomography of \tilde{H}_A

what is the operator structure of \tilde{H}_A ? ... *in quantum simulation*

Basic idea: We make an *ansatz* for \tilde{H}_A with *quasi-local few-body terms*, which we fit to data, and verify with few measurements



arXiv:2009.09000

On the quasi-local ansatz for the Entanglement Hamiltonian

Hamiltonians H of physical systems: as sum of quasi-local few-body terms

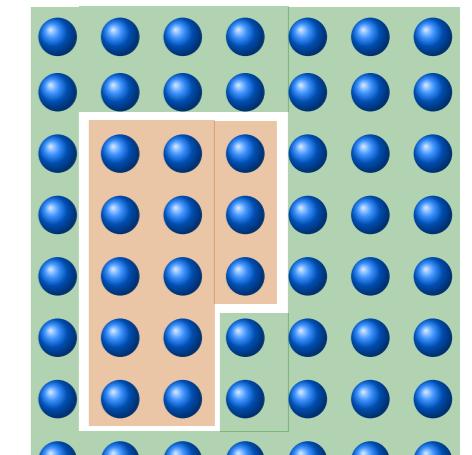
Example:

$$H = \sum_{i < j} \left(J_{ij} \sigma_i^+ \sigma_j^- + \text{h.c.} \right) + B \sum_i \sigma_i^z$$

few parameters

$$\text{with } J_{ij} = \frac{J_0}{|i-j|^\eta} \text{ for } B \gg |J_{ij}|$$

bipartition $A:B$



$|\Psi\rangle$

Structure of Entanglement Hamiltonian \tilde{H}_A : quasi-local few-body terms?

example: thermal state from quench

$$\text{Tr}_B |\Psi_t\rangle\langle\Psi_t| \longrightarrow \rho_A \sim e^{-\beta H_A}$$

system Hamiltonian on A

Bisognano-Wichmann theorem (QFT)

- for ground states

Conformal Field Theory

- in quench dynamics, ground states

heuristic approach:
test & verify

Suggests ansatz for \tilde{H}_A as deformation of system Hamiltonian $H_A \equiv H|_A + \text{corrections (new physics?)}$

On the quasi-local ansatz for the Entanglement Hamiltonian

Hamiltonians H of physical systems: as sum of quasi-local few-body terms

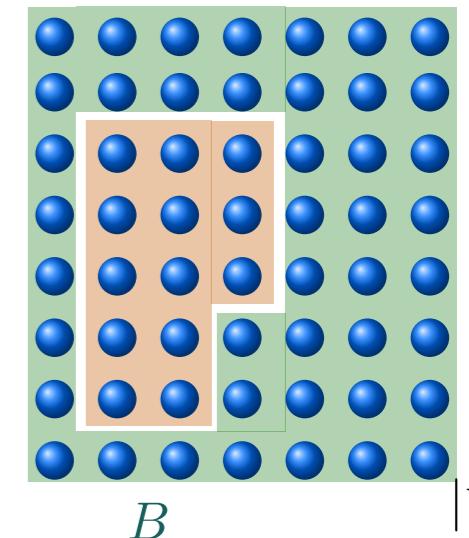
Example:

$$H = \sum_{i < j} \left(J_{ij} \sigma_i^+ \sigma_j^- + \text{h.c.} \right) + B \sum_i \sigma_i^z$$

few parameters

$$\text{with } J_{ij} = \frac{J_0}{|i-j|^\eta} \text{ for } B \gg [J_{ij}]$$

bipartition $A:B$



Structure of Entanglement Hamiltonian \tilde{H}_A : quasi-local few-body terms?

Ansatz:

$$\tilde{H}_A(\tilde{\mathbf{J}}, \tilde{\mathbf{B}}) = \sum_{ij} \left(\tilde{J}_{ij} \sigma_i^+ \sigma_j^- + \text{h.c.} \right) + \sum_i \tilde{B}_i \sigma_i^z + \sum_{ij} \tilde{B}_{ij} \sigma_i^z \sigma_j^z + \sum_{ijk} \left(\tilde{J}_{ijk} \sigma_i^+ \sigma_j^- \sigma_k^z + \text{h.c.} \right)$$

few variational parameters

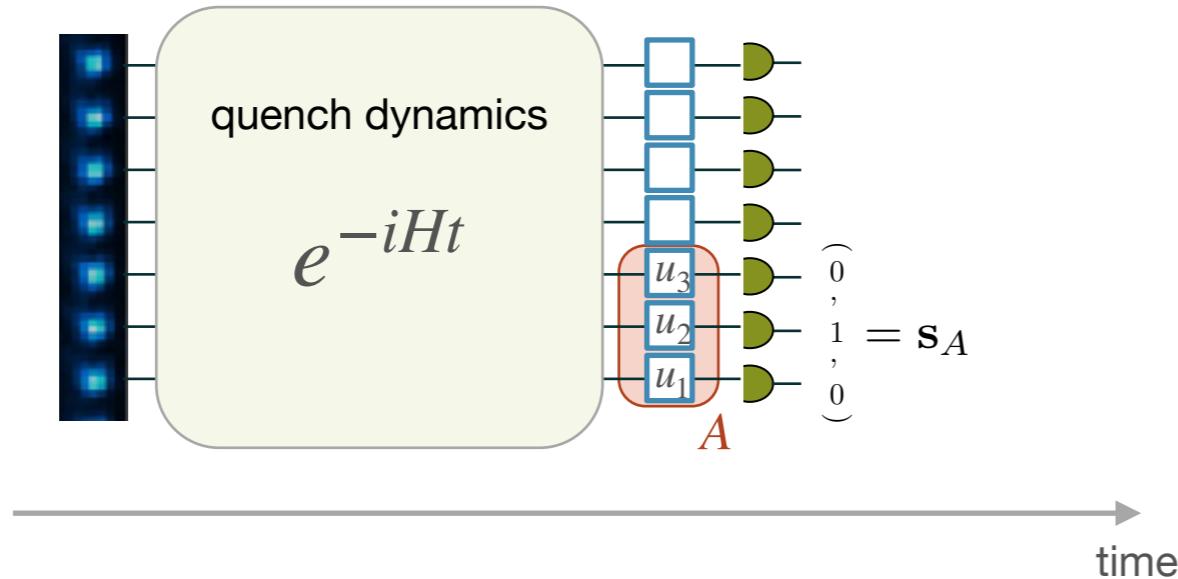
deformed system Hamiltonian (base level)

corrections (level 1)

Suggests \tilde{H}_A as deformation of system Hamiltonian $H_A \equiv H|_A + \text{corrections (new physics?)}$

Entanglement Hamiltonian Tomography (EHT)

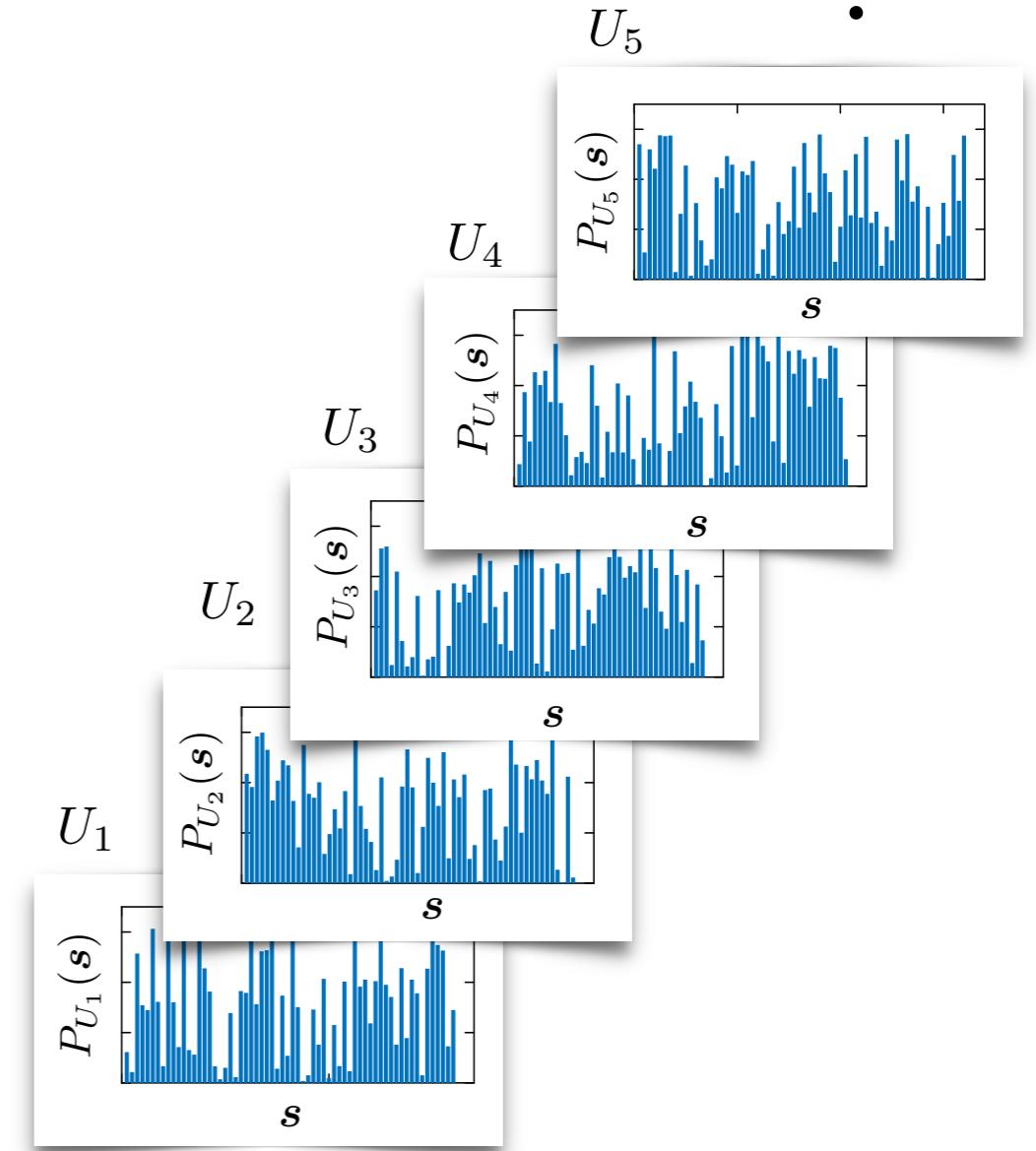
Here: from randomized local unitaries



Protocol:

- Ansatz for $\tilde{H}_A(\tilde{\mathbf{g}})$, which is physically motivated
- Best fit to experimental observations: $\tilde{\mathbf{g}}$

$$\chi^2(\tilde{\mathbf{g}}) = \sum_{U,s} \left[\text{Tr} \left(U |s\rangle\langle s| U^\dagger \frac{\exp(-\tilde{H}_A(\tilde{\mathbf{g}}))}{Z(\tilde{\mathbf{g}})} \right) - P_U(s) \right]^2$$

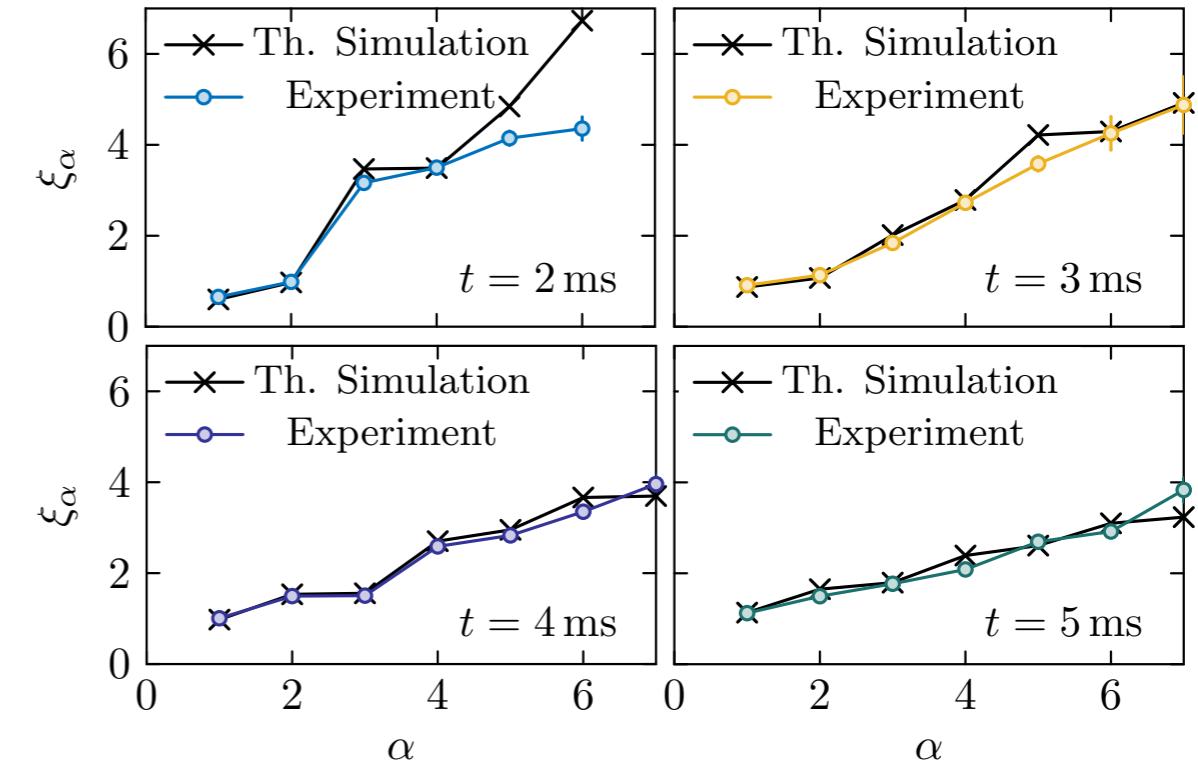
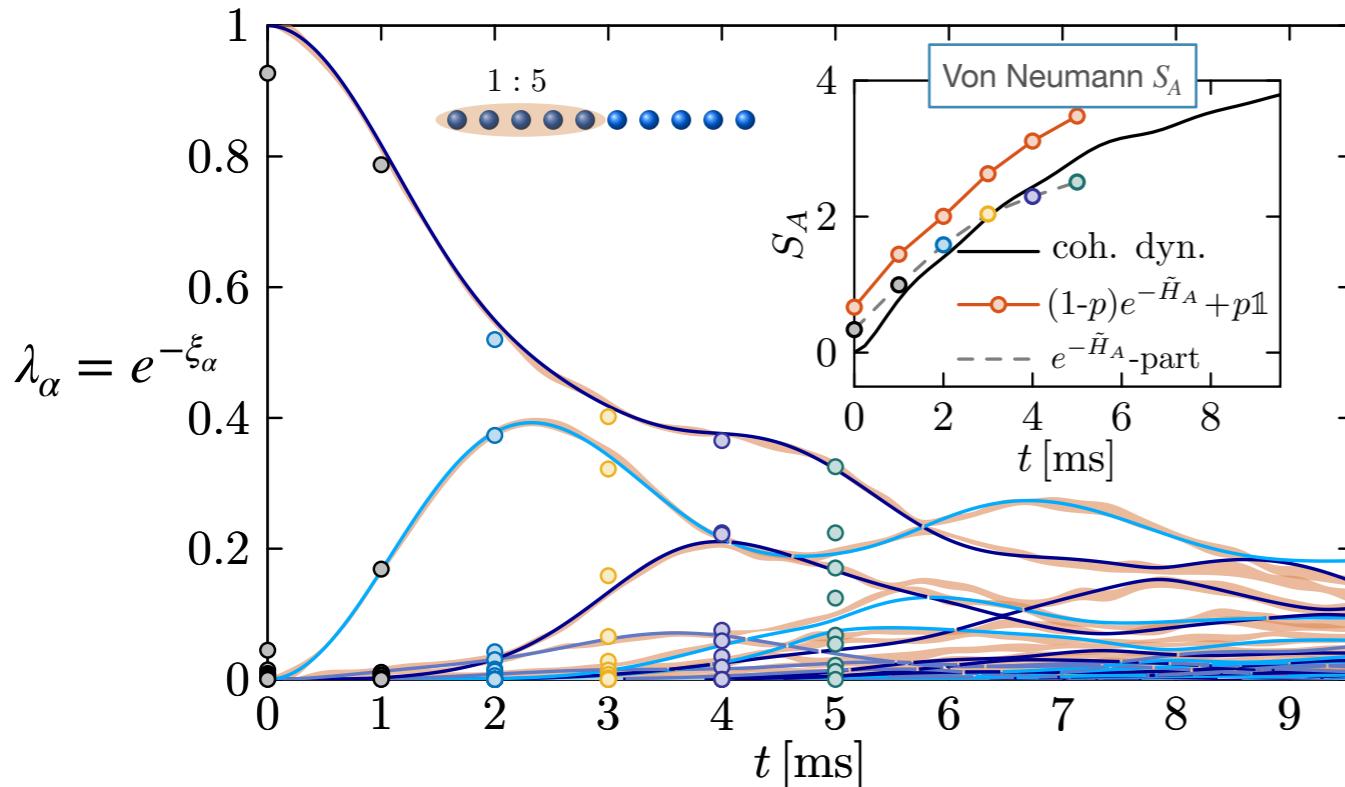


Experiment: EH-Fit for a quench with 10 ions (sub-system 1:5)



C Kokail

Seeing Schmidt decomposition *live*: $|\Psi\rangle = \sum_{\alpha=1}^{\chi_A} \lambda_\alpha |\Phi_\alpha^A\rangle \otimes |\Phi_\alpha^B\rangle$ from data published in Brydges et al., Science 2019



Ansatz for the reduced density matrix
with p decoherence parameter

$$\rho_A(\tilde{\mathbf{g}}) = (1 - p)e^{-\tilde{H}_A(\tilde{\mathbf{g}})} + p \frac{\mathbb{1}}{\mathcal{D}_A}$$

$$\begin{aligned} \eta &\simeq 1.2 \\ N_U &= 500 \quad N_M = 150 \end{aligned}$$

... and similar results for 20 ions and (sub-system 8:14)



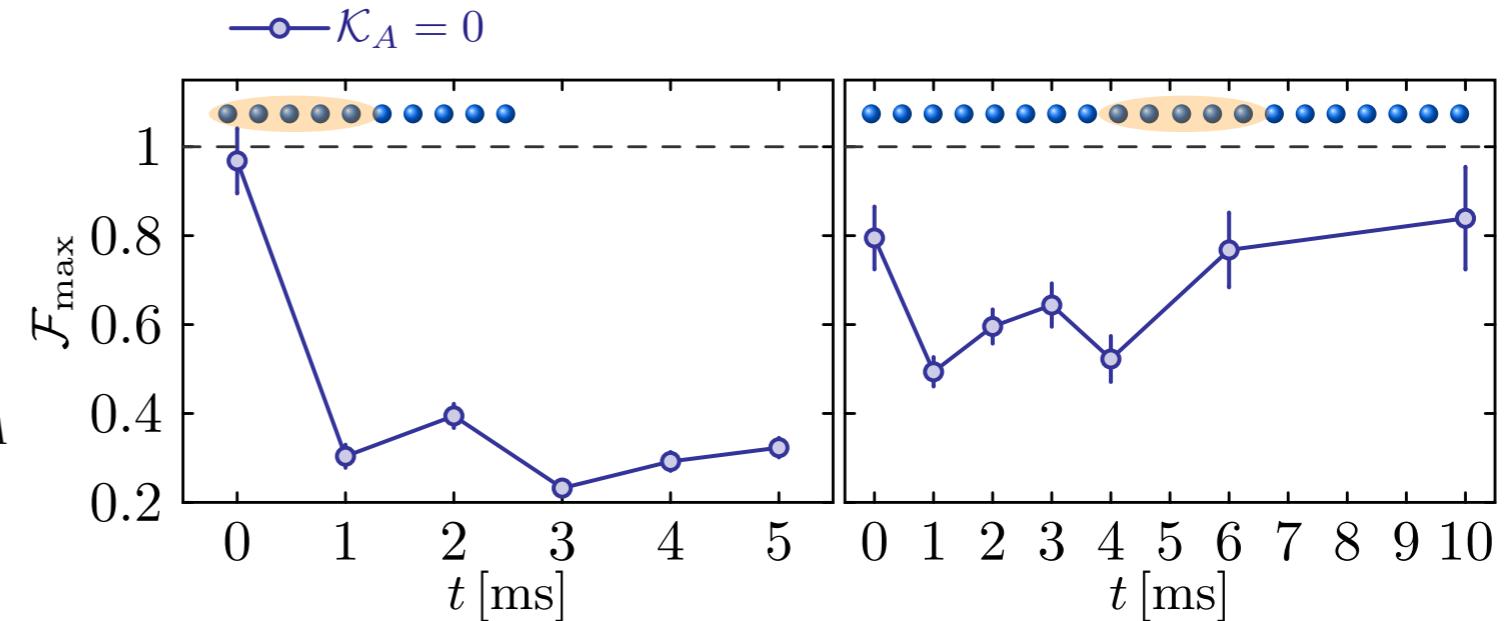
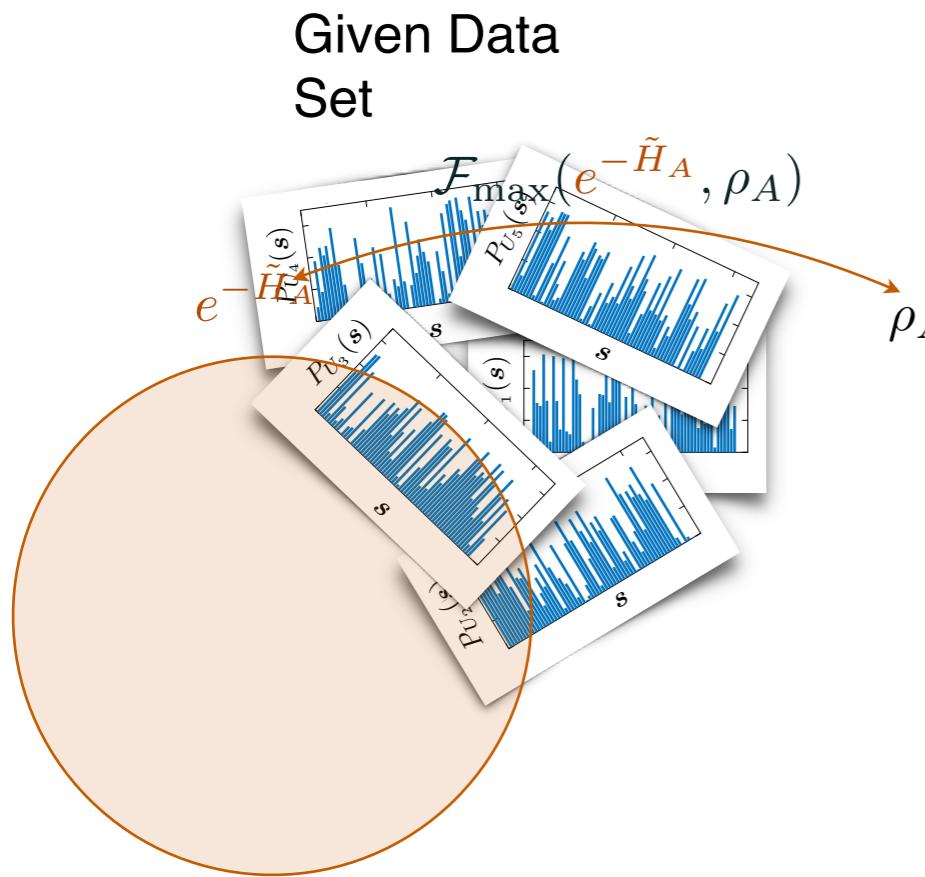
C Kokail

Testing the Ansatz via Fidelity Estimation

$$\mathcal{F}_{\max}(\rho_1, \rho_2) = \frac{\text{Tr}[\rho_1 \rho_2]}{\max\{\text{Tr}[\rho_1^2], \text{Tr}[\rho_2^2]\}}$$

see cross-platform verification of NISQ

A.Elben et.al.
Phys. Rev. Lett. 124, 010504



$$\tilde{H}_A = \sum_{i,j < i} \tilde{J}_{ij} (\sigma_i^+ \sigma_j^- + \text{H.c.}) + \sum_i \tilde{B}_i \sigma_i^z$$



C Kokail

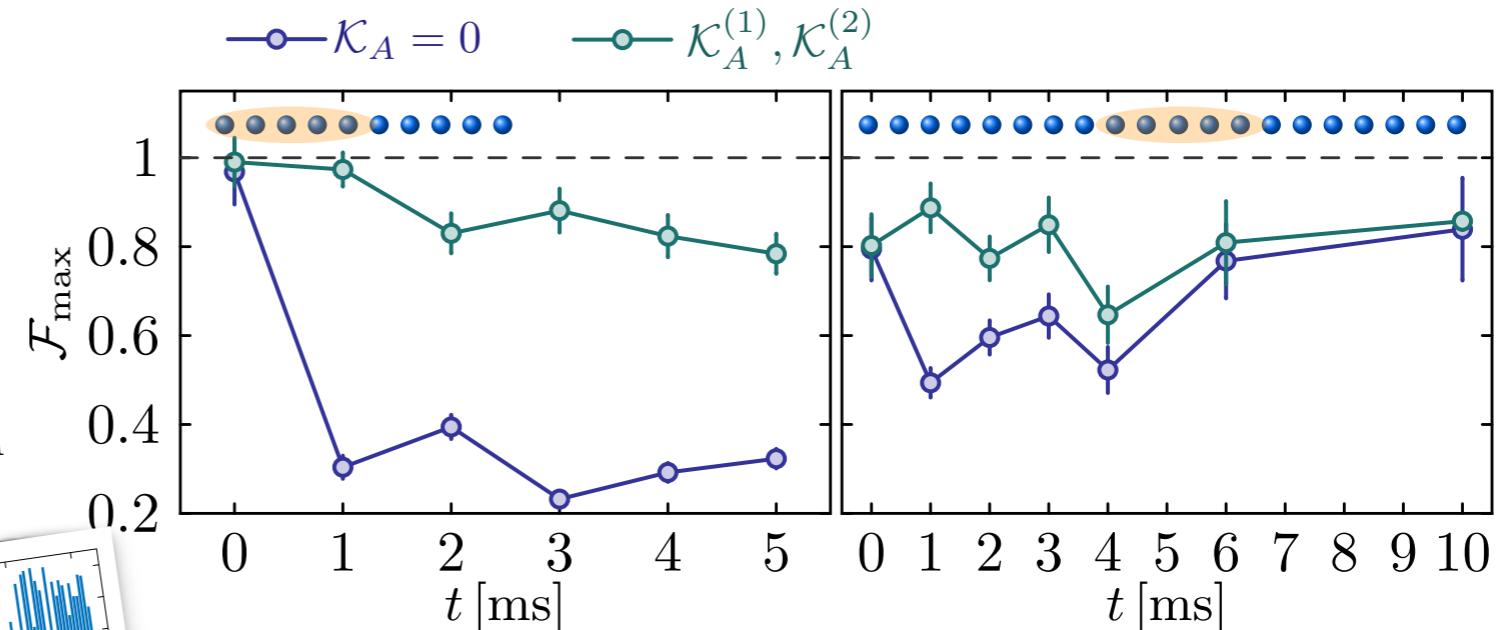
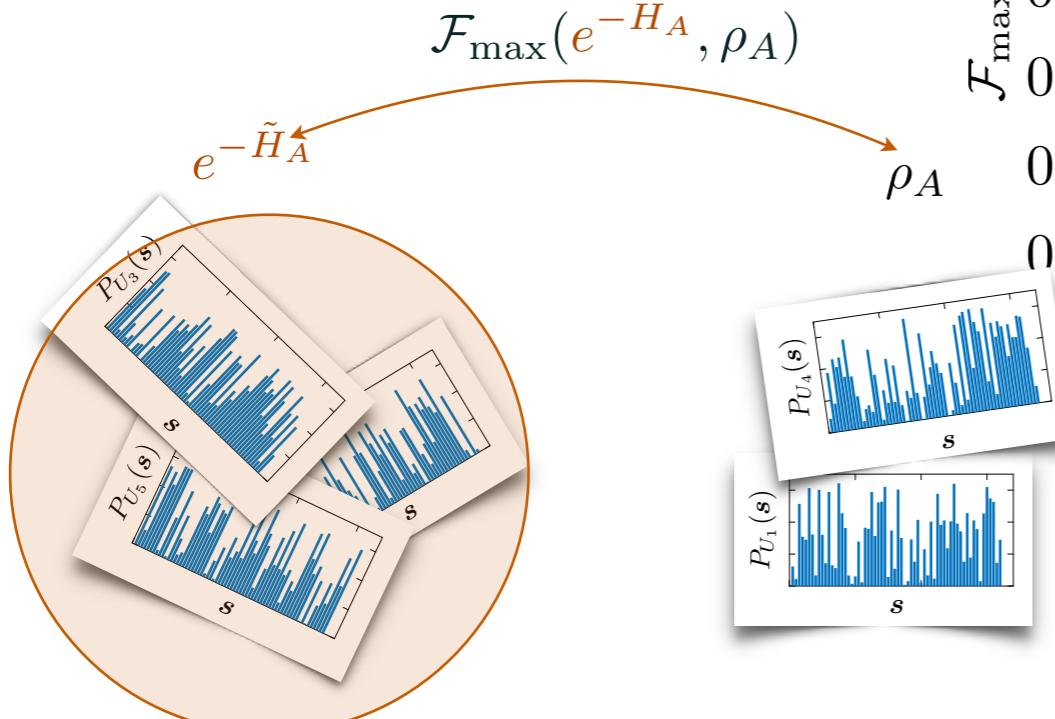
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A.Elben et.al.
Phys. Rev. Lett. 124, 010504

Given Data
Set



$$\mathcal{K}_A^{(1)} = \sum_{k < l \in A} \tilde{J}_{kl}^{XY} (\sigma_k^x \sigma_l^y - \sigma_k^y \sigma_l^x)$$

$$\mathcal{K}_A^{(2)} = \sum_{k < l} \sum_{m \neq k, l} \tilde{J}_{klm}^{XYZ} (\sigma_k^x \sigma_l^y \sigma_m^z - \sigma_k^y \sigma_l^x \sigma_m^z)$$



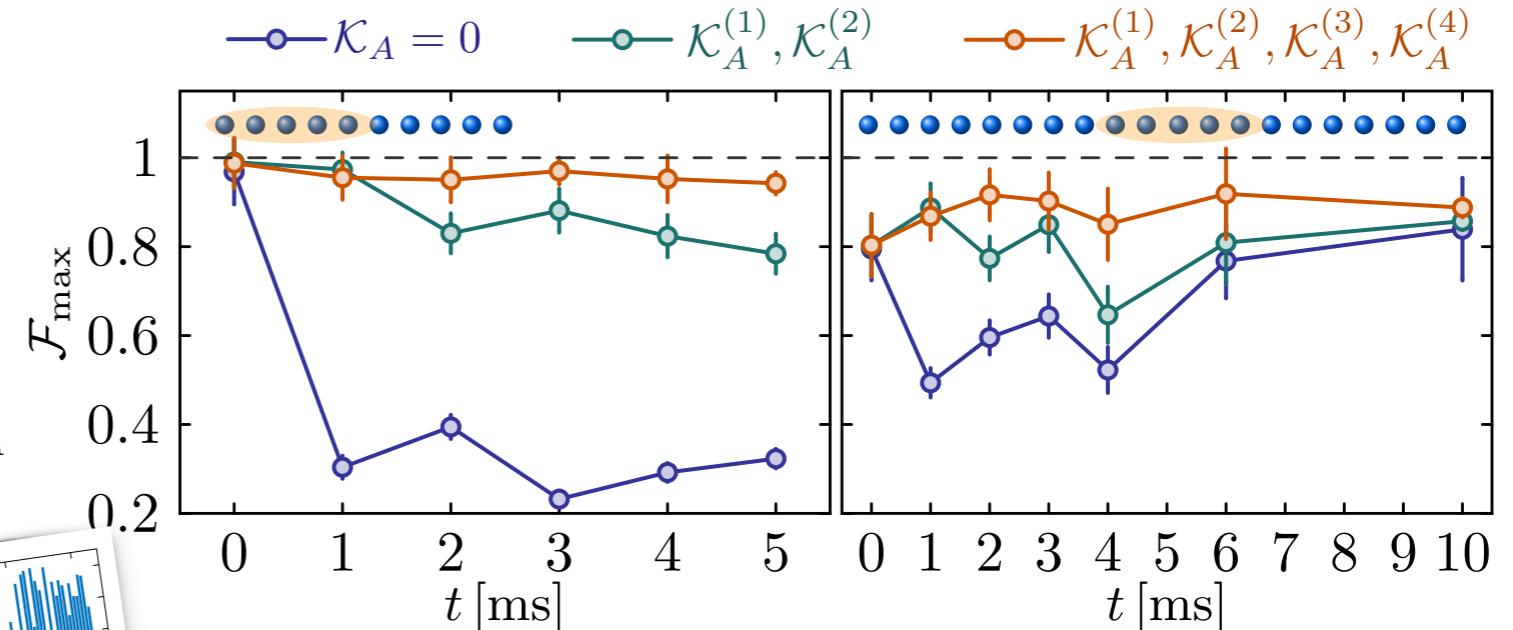
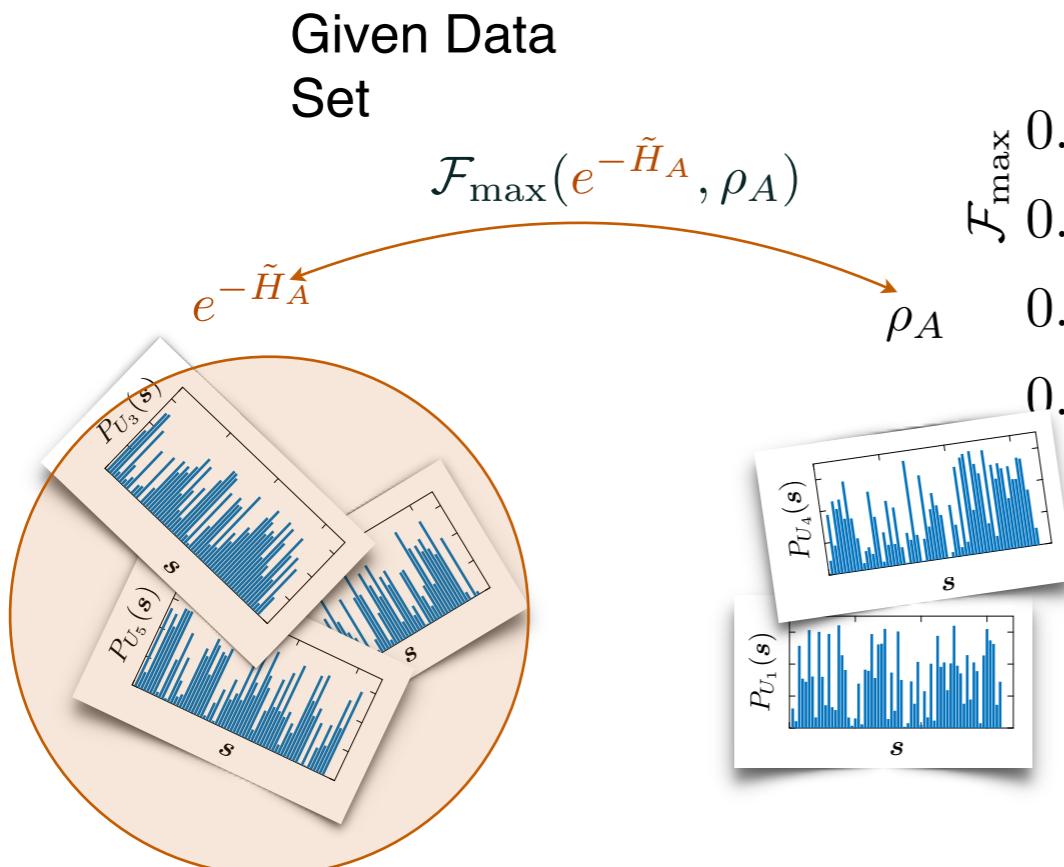
C Kokail

Testing the Ansatz via Fidelity Estimation

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see cross-platform verification of NISQ

A.Elben et.al.
Phys. Rev. Lett. 124, 010504



$$\mathcal{K}_A^{(3)} = \sum_{k < l \in A} \tilde{J}_{kl}^{ZZ} \sigma_k^z \sigma_l^z + \sum_{k < l < m \in A} \tilde{J}_{klm}^{ZZZ} \sigma_k^z \sigma_l^z \sigma_m^z$$

$$\mathcal{K}_A^{(4)} = \sum_{k < l} \sum_{m \neq k, l} \tilde{J}_{klm}^{XXZ} (\sigma_k^x \sigma_l^x \sigma_m^z + \sigma_k^y \sigma_l^y \sigma_m^z)$$

Q.: Why entanglement Hamiltonian
quasi-local and few-body?

Q.: Why entanglement Hamiltonian
quasi-local and few-body?



M. Dalmonte P. Calabrese
→ ICTP SISSA

thanks for insights and discussions

Bisognano-Wichmann Theorem of QFT

Conformal Field Theory

predict quasi-local entanglement Hamiltonian

- for ground states (BW+CFT)
 - quench dynamics 1+1D (CFT) to critical point

validity, assumptions: continuum \rightarrow lattice, ... ?

M. Dalmonte et al., Nat. Phys. (2018); G. Giudici et al. PRB (2018).

P. Calabrese & J. Cardy, J Phys A (2009)

X Wen, S Ryu & AW Ludwig, J Stat Mech (2018).

Quantum simulation as testbed for lattice-BW & CFT

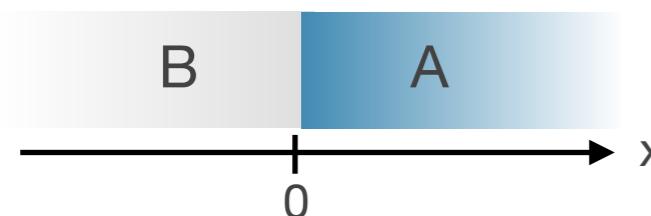
Potential to ‘discover’ new physics

Bisognano-Wichmann Theorem [ground states]

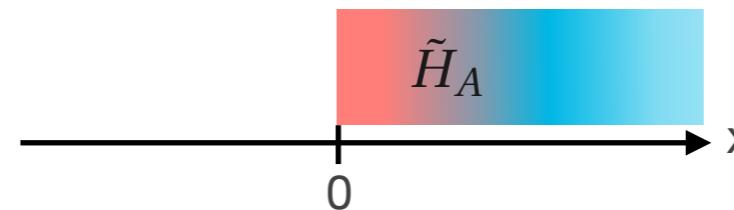


M. Dalmonte
→ ICTP

physical system

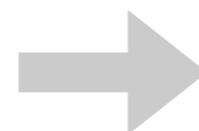


entanglement Hamiltonian



Hamiltonian density (here 1D)

$$H(x)$$



Lorentz invariance

BW Entanglement Hamiltonian (here 1D)

$$\rho_A \sim e^{-\int_{x>0} dx \beta(x) H(x)}$$

with ‘temperature’ $\beta(x) \sim x$

$$\tilde{H}_A \sim \int_{x>0} dx [x H(x)]$$

\uparrow
 $\tilde{H}_A(x)$

EH as local, few-body Hamiltonian with *spatially dependent* couplings

Lattice models?

Numerical examples show that this works well in many cases

M. Dalmonte et al., Nat. Phys. (2018).
G. Giudici et al. PRB (2018).

Can our Entanglement Hamiltonian Tomography *measure* BW?

EHT applied to ground states ... *in light of Bisognano-Wichmann on lattice*

System Hamiltonian

$$H = \sum_{i,j>i} \frac{J_0}{|i-j|^\eta} \sigma_i^x \sigma_j^x + B \sum_i \sigma_i^z$$

$\eta = 2.5$

Ansatz for the entanglement Hamiltonian:

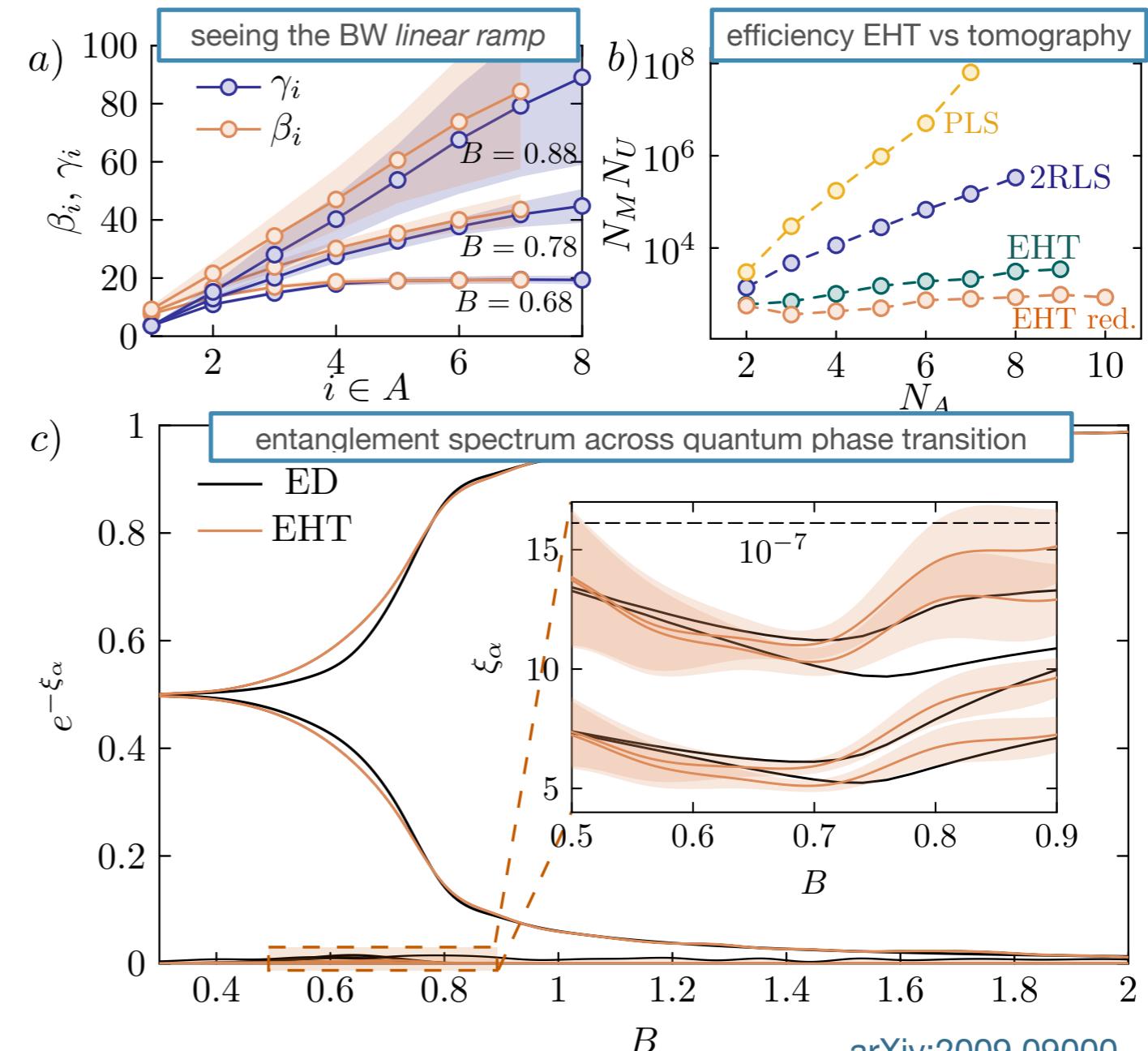
$$\tilde{H}_A = \sum_{i,j \in A} \tilde{J}_{ij} \sigma_i^x \sigma_j^x + \sum_{i \in A} \tilde{B}_i \sigma_i^z$$

where we choose a parabolic deformation

$$\tilde{J}_{ij} = \frac{\vartheta_0 + \vartheta_1(i+j) + \vartheta_2(i+j)^2}{2|i-j|^\alpha}$$

$$\tilde{B}_i = B(\vartheta_0 + \vartheta_1 i + \vartheta_2 i^2)$$

We *simulate* EHT measurement runs



Revealing new physics in quench dynamics

CFT predicts corrections to EH as ‘distortion of the system Hamiltonian’. Can we detect them?

We simulate a global quench in the transverse field Ising model ($\eta \rightarrow \infty$)

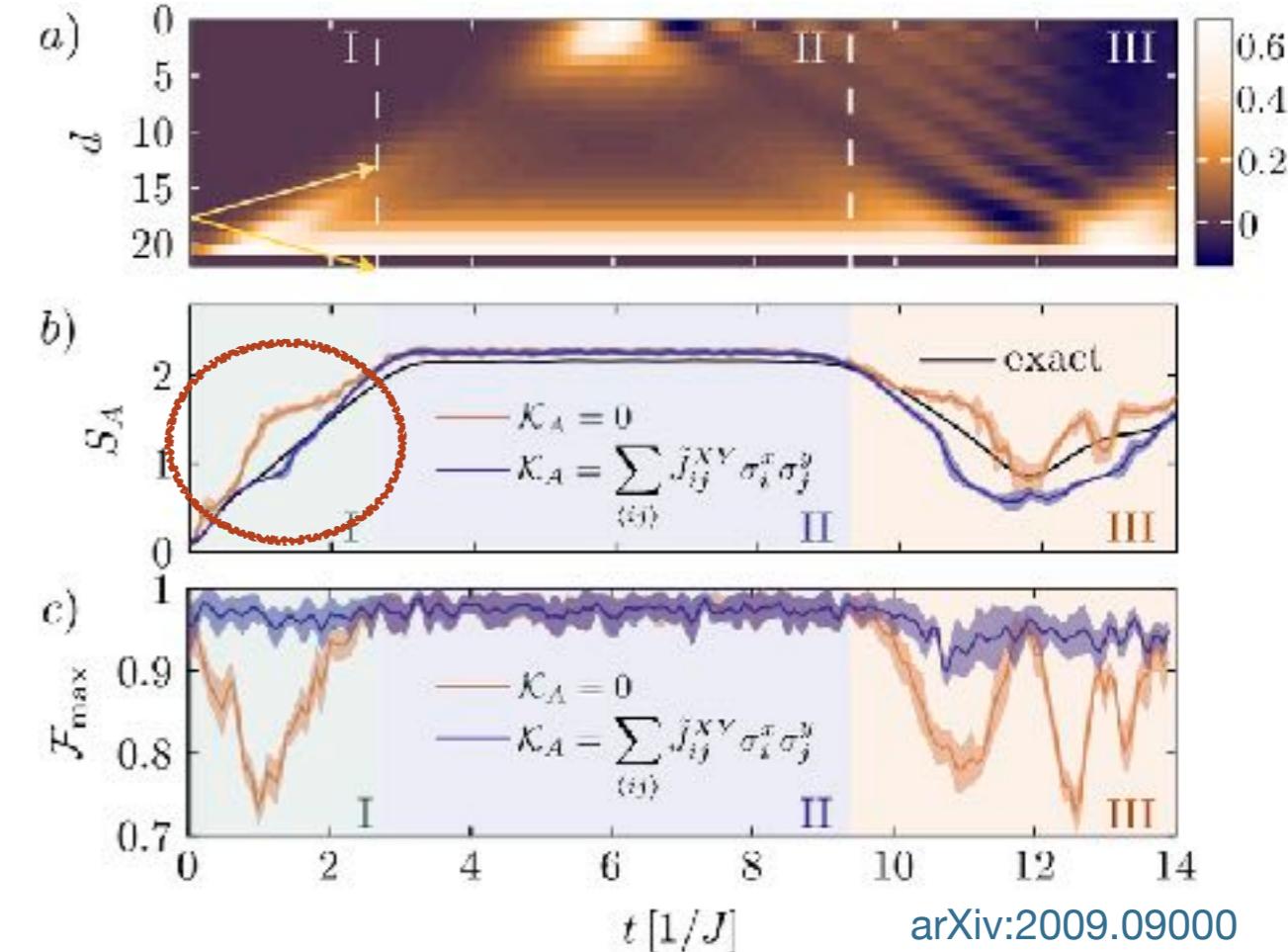
Ansatz for the entanglement Hamiltonian:

$$\tilde{H}_A = \sum_{i \in A} \left(\tilde{J}_{i,i+1} \sigma_i^x \sigma_{i+1}^x + \tilde{B}_i \sigma_i^z \right) + \mathcal{K}_A$$

with \mathcal{K}_A motivated from the local lattice momenta

$$\mathcal{K}_A = \sum_{i \in A} \tilde{J}_{i,i+1}^{XY} \sigma_i^x \sigma_{i+1}^y$$

We *simulate* EHT measurement runs



J. Cardy and E. Tonni, J. Stat. Mech. 2016

X Wen, S Ryu & AW Ludwig, J Stat Mech (2018)

W. Zhu, Z. Huang, Y.-C. He, & X. Wen, PRL (2020)

Conclusions and Outlook

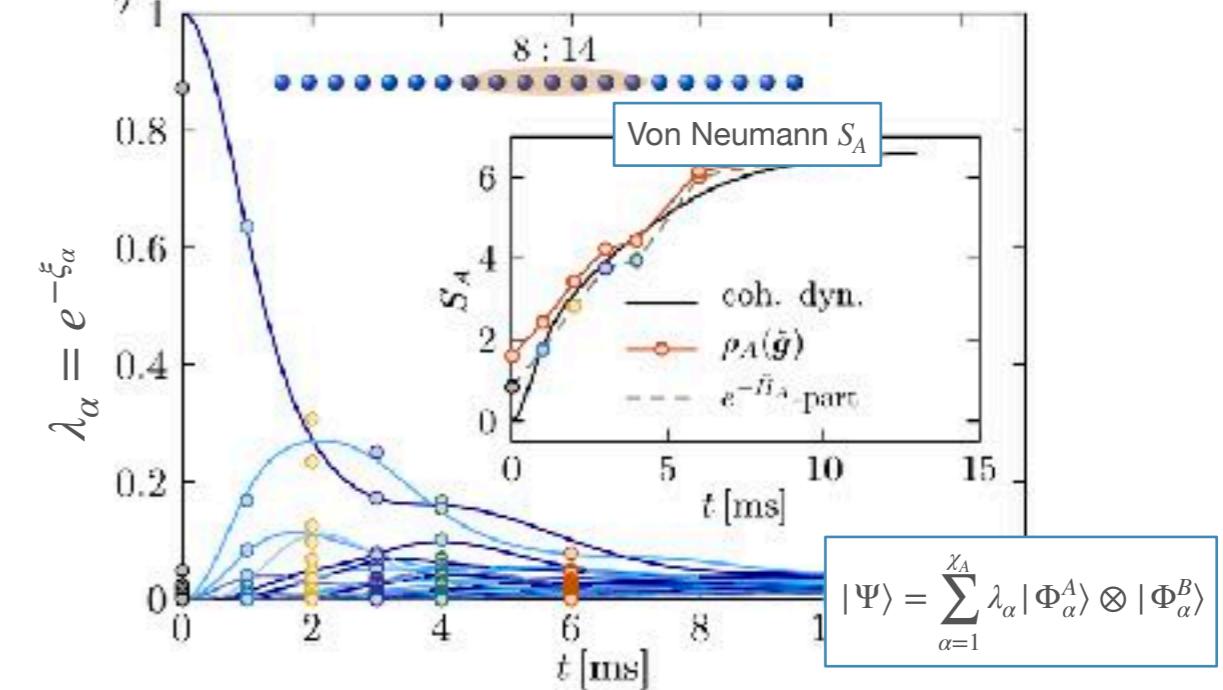
- Revealing and quantifying entanglement as a key challenge in quantum simulation

Randomized measurement toolbox

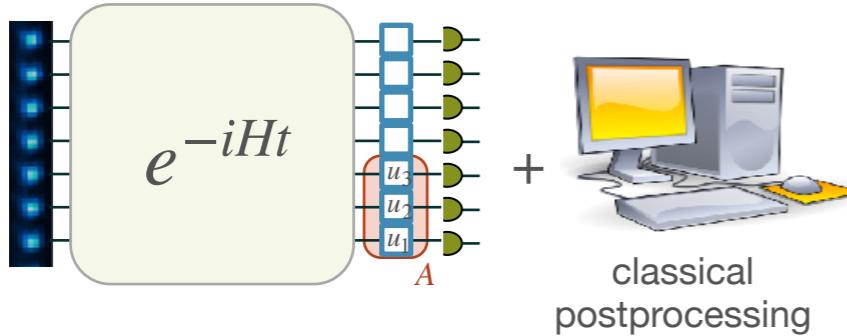
Protocols: from entanglement entropies to entanglement Hamiltonian tomography

Tailored to today's quantum simulation platforms

Schmidt spectrum for quench: 20-ion quantum simulator



this talk:



measure / characterize / quantify entanglement in quantum simulation

Entanglement Hamiltonian Tomography

- ✓ heuristic
- ✓ testbed for BW & CFT

Conclusions and Outlook

- Revealing and quantifying entanglement as a key challenge in quantum simulation

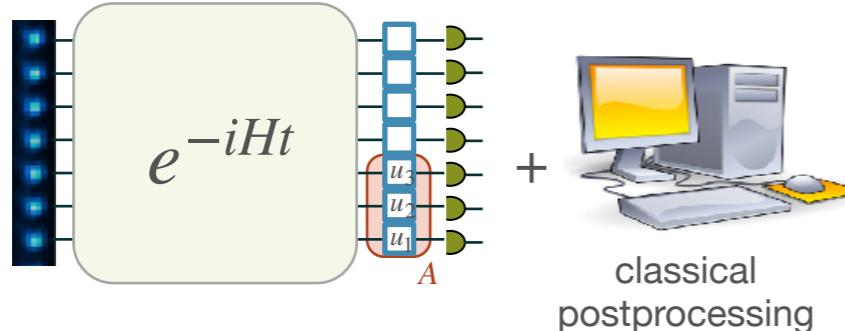
Randomized measurement toolbox

Protocols: from entanglement entropies to entanglement Hamiltonian tomography

Tailored to today's quantum simulation platforms

- Future - *quantum protocols?*

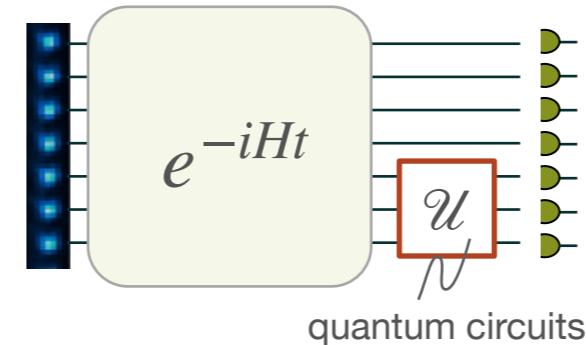
this talk:



more efficient? in quantum supremacy regime/large A ?

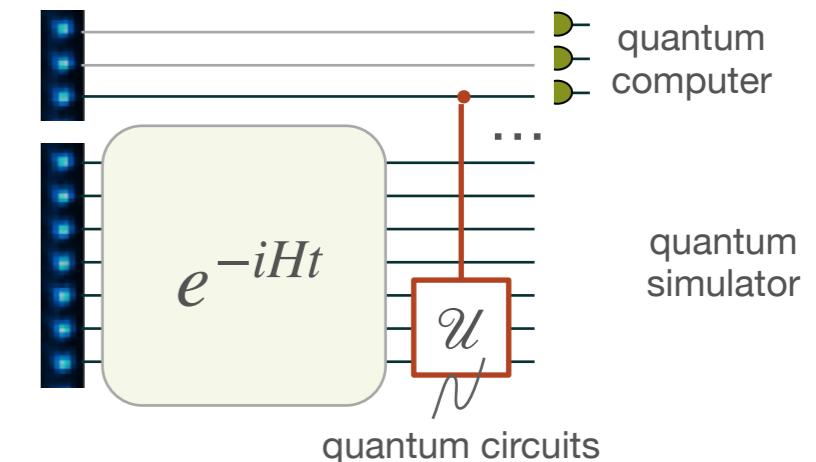
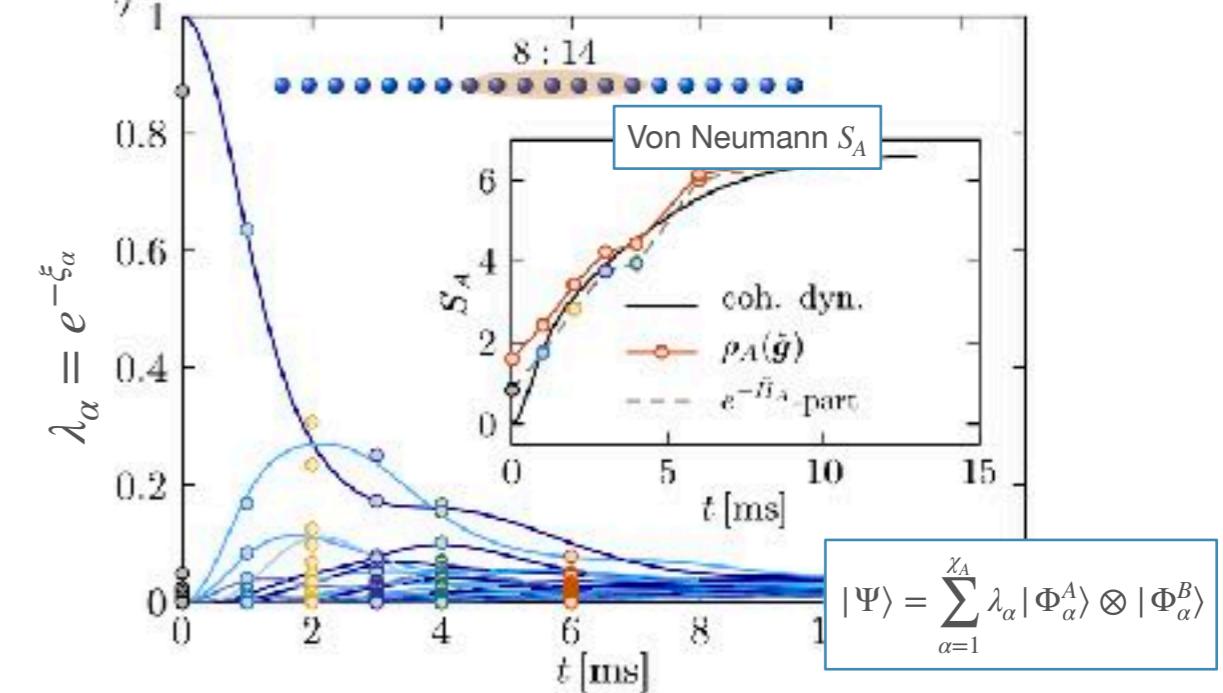
quantum simulators in 50 ~ few hundred regime

towards quantum algorithms



Bhuvanesh Sundar et al., unpublished

Schmidt spectrum for quench: 20-ion quantum simulator



D Vasilyev, A Grankin et al., PRXQ in press

present group members



A Elben



B Vermersch
→ Grenoble



C Kokail



R. van Bijnen



B Sundar



D Vasilyev



L Joshi



Jinlong Yu



M Baranov
(senior member)



R Kaubrügger



A Kruckenhauser



T Olsacher



T Zache



W Hahn



M Di Liberto

former group members involved in these / related projects



M. Dalmonte
→ ICTP



A Grankin
→ JQI



L Sieberer
→ UIBK



P Hauke
→ Trento



H Heyl
→ Dresden

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