Quantum Verification

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Quantum DRAMA

A powerful quantum device

Vs

Verifier (with restricted computational power)



Quantum DRAMA

Quantum Advantage



Quantum Devices Noise



Quantum Verification



Quantum Verification



- Powerful quantum server(s)
- Certify the correctness of the computation









Completeness:
$$(\forall x \in L) \operatorname{Pr} [(V \leftrightarrow P)(x) \ accepts] = 1$$



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Soundness:
$$(\forall x \notin L)(\forall P') \operatorname{Pr}\left[(V \leftrightarrow P')(x) \ accepts\right] \leq \frac{1}{2}$$

BPP as Interactive Proof System



Yes *X* satisfies some property



























Towards Practical Verification

Gottesman (04) - Vazirani (07) - Aaronson \$25 Challenge (07)

Does BQP admit a quantum interactive protocol where the honest prover is in BQP and the verifier is in BPP?

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Does BQP admit a quantum interactive protocol where the honest prover is in BQP and the verifier is in BPP?

Yes, if verifier can prepare some random qubits

Yes, if provers are entangled but none-communicating

Yes, if malicious prover cannot break LWE

Aharonov, Ben-Or, and Eban, ICS 2010 Broadbent, Fitzsimons and Kashefi, FOCS 2009

Reichardt, Unger, Vazirani, Nature 2013

Mahadev, FOCS 2018





• Correctness: in the absence of any *interference/noise/deviation*, verifier accepts and the computation output is correct

• Soundness: Verifier rejects an incorrect output, except with probability at most exponentially small in the security parameter



Protocol	Verifier resources	Communication	2-way quantum comm.
Clifford-QAS VQC	$O(log(1/\epsilon))$	$O(N \cdot log(1/\epsilon))$	Y
Poly-QAS VQC	$O(log(1/\epsilon))$	$O((n+L) \cdot log(1/\epsilon))$	N
VUBQC	O(1)	$O(N \cdot log(1/\epsilon))$	N
Test-or-Compute	O(1)	$O((n+T) \cdot log(1/\epsilon))$	Ν

x the input to this circuit, then $n = |x|, N = |\mathcal{C}|$

Single-prover receive-and-measure

verifier receives quantum states from the prover and has the ability to measure them

- Post-hoc Verification (none hiding)
- Measuring only blind QC

Protocol	Measurements	Observables	Blind
Measurement-only	$O(N \cdot 1/lpha \cdot 1/\epsilon^2)$	5	Y
Hypergraph measurement-only	$O(max(N, 1/\epsilon^2)^{22})$	3	Y
1S-Post-hoc	$O(N^2 \cdot log(1/\epsilon))$	2	Ν
Steering-based VUBQC	$O(N^{13}log(N) \cdot log(1/\epsilon))$	5	Y



Protocol	Provers	Qmem provers	Rounds	Communication	Blind
RUV	2	2	$O(N^{8192} \cdot log(1/\epsilon))$	$O(N^{8192} \cdot log(1/\epsilon))$	Y
McKague	$O(N^{22} \cdot log(1/\epsilon))$	0	$O(N^{22} \cdot log(1/\epsilon))$	$O(N^{22} \cdot log(1/\epsilon))$	Y
GKW	2	1	$O(N^{2048} \cdot log(1/\epsilon))$	$O(N^{2048} \cdot log(1/\epsilon))$	Y
HPDF	$O(N^4 log(N) \cdot log(1/\epsilon))$	$O(log(1/\epsilon))$	$O(N^4 log(N) \cdot log(1/\epsilon))$	$O(N^4 log(N) \cdot log(1/\epsilon))$	Y
\mathbf{FH}	5	5	$O(N^{16} \cdot log(1/\epsilon))$	$O(N^{19} \cdot log(1/\epsilon))$	Ν
NV	7	7	O(1)	$O(N^3 \cdot log(1/\epsilon))$	Ν

Prepare and Send Verification



Soundness

Verifier

 ν

random parameters

Soundness

Verifier Prover/Device

 ν

random parameters









For any

Prover's deviation/cheating strategy (Any noise/failure model)

the probability of verifier accepting an incorrect outcome density operator is bounded by ϵ :



$$P_{incorrect}^{\nu} = \left(\mathbb{I} - |\Psi_{ideal}^{\nu}\rangle \left\langle \Psi_{ideal}^{\nu}|\right) \otimes |r_{t}^{\nu}\rangle \left\langle r_{t}^{\nu}\right|$$
Accept Key



Accept Key

 $\sum_{\nu} p(\nu) \ Tr\left(P_{incorrect}^{\nu} B(\nu)\right) \leq \epsilon$

Test or Compute








Reduction



Reduction



cryptography reduces the **verification** problem to **error-detection** procedure bypassing the complexity of simulation

Cryptography Toolkit



Broadbent, Fitzsimons and Kashefi, FOCS09

Cryptography Toolkit



Universal Blind Quantum Computing: QKD + Teleportation

Broadbent, Fitzsimons and Kashefi, FOCS09





Verifiable Universal Blind Quantum Computing: QKD + Teleportation + Test



















$$B_{j}(\nu) = \operatorname{Tr}_{B}\left(\sum_{b} |b + c_{r}\rangle \langle b| C_{\nu_{C},b} \Omega \mathcal{P}((\otimes^{B} |0\rangle \langle 0|) \otimes |\Psi^{\nu,b}\rangle \langle \Psi^{\nu,b}|) \mathcal{P}^{\dagger} \Omega^{\dagger} C_{\nu_{C},b}^{\dagger} |b\rangle \langle b + c_{r}|\right)$$



$$B_{j}(\nu) = \operatorname{Tr}_{B}\left(\sum_{b} |b + c_{r}\rangle \langle b| C_{\nu_{C},b}\Omega \mathcal{P}((\otimes^{B} |0\rangle \langle 0|) \otimes |\Psi^{\nu,b}\rangle \langle \Psi^{\nu,b}|) \mathcal{P}^{\dagger}\Omega^{\dagger}C_{\nu_{C},b}^{\dagger} |b\rangle \langle b + c_{r}|\right)$$



$$B_{j}(\nu) = \operatorname{Tr}_{B}\left(\sum_{b} \left|b + c_{r}\right\rangle \left\langle b\right| C_{\nu_{C},b} \Omega \mathcal{P}(\left(\otimes^{B} \left|0\right\rangle \left\langle 0\right|\right) \otimes \left|\Psi^{\nu,b}\right\rangle \left\langle\Psi^{\nu,b}\right|\right) \mathcal{P}^{\dagger} \Omega^{\dagger} C_{\nu_{C},b}^{\dagger} \left|b\right\rangle \left\langle b + c_{r}\right|\right)$$

Soundness Proof

 $p_{incorrect} = \sum_{\nu} p(\nu)' \operatorname{Tr}(P_{\perp} \otimes |\eta_t^{\nu}\rangle \langle \eta_t^{\nu}| \ (\Omega \mathcal{P}((\otimes^B |0\rangle \langle 0|) \otimes |\Psi^{\nu}\rangle \langle \Psi^{\nu}|) \mathcal{P}^{\dagger}\Omega^{\dagger})).$













Summary











(a) Trap-colouring

Kashefi, Walden, 2014



The challenge of fault tolerant verification

Aharonov, Ben-Or, Eban, Mahadev, 2015

Linear Server overhead

Linear Server overhead

Inverse-polynomial security

Linear Server overhead



Inverse-polynomial security

Bootstrapping via Fault Tolerance encoding

Linear Server overhead



Inverse-polynomial security

Bootstrapping via Fault Tolerance encoding

Huge Server overhead


New Result

Leichtle, Music, Kashefi, Ollivier, PRX, 2021

BQP: Classical Input/Output

New Result

Leichtle, Music, Kashefi, Ollivier, PRX, 2021

No Server overhead

BQP: Classical Input/Output

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BQP: Classical Input/Output

BQP: Classical Input/Output New Result Leichtle, Music, Kashefi, Ollivier, PRX, 2021 No Server overhead Inverse-polynomial security Classical Repetition Code





Prepare and Send Verification



Target computation



Blind Target computation



Insertion of Trap



Insertion of Trap



Test and Compute

n := t + d

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n := t + d

Verifier counts the number of failed test rounds If > threshold *w*, aborts

Otherwise accepts the majority outcome of the computation rounds as output

Protocol Features

Protocol Features Redo Option Verifier or Server may experience unintentional devices failures



quantum attacks entangled across rounds are much more powerful than what classical correlations allow



Fine-Tuning the Number of Repetitions

Small *k*-colouring of the graph

Protocol Features Redo Option Verifier or Server may experience unintentional devices failures **Exponential Security Amplification** quantum attacks entangled across rounds are much more powerful than what classical correlations allow

Fine-Tuning the Number of Repetitions

Small *k*-colouring of the graph

Composable Security

Abstract Cryptography

Security Proof

Theorem 1 (Security of Protocol 1). For n = d+t such that d/n and t/n are fixed in (0, 1) and w such that w/t is fixed in $(0, \frac{1}{k} \cdot \frac{2p-1}{2p-2})$, where p is the inherent error probability of the BQP computation, Protocol 1 with d computation rounds, t test rounds, and a maximum number of tolerated failed test rounds of w is ϵ -composably-secure with ϵ exponentially small in n.

Robustness

On honest (but possibly noisy) devices, the protocol accepts with high probability

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On honest (but possibly noisy) devices, the protocol accepts with high probability

- The noise can be modelled by round-dependent Markovian processes – i.e. a possibly different arbitrary CPTP map acting on each round.
- The probability that at least one of the trap measurements fails in any single test round is upperbounded by some constant $p_{max} < \frac{1}{k} \cdot \frac{2p-1}{2p-2}$ and lower-bounded by $p_{min} \leq p_{max}$.

Summary

Decoupling Verifiability and Fault-Tolerance

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Decoupling Verifiability and Fault-Tolerance

all qubits can be devoted to useful computations irrespective of the desired security

the average ratio of failed test rounds to be upper-bounded





- Online vs. offine
- Device-independent vs. one-sided device-independent
- I.I.D. states vs. general states
- Privacy preserving vs non-hiding
- Universal vs non-universal
- And many others

Eisert, Hangleiter, Walk, Roth, Markham, Parekh, Chabaud, Kashefi

Enforcing the *correct functioning* of a quantum device, using the minimum amount of *resources*, while making as few *assumptions* as possible.

Eisert, Hangleiter, Walk, Roth, Markham, Parekh, Chabaud, Kashefi



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Figures of merit

Enforcing the *correct functioning* of a quantum device, using the minimum amount of *resources*, while making as few *assumptions* as possible.

User/Device tunability resources

Eisert, Hangleiter, Walk, Roth, Markham, Parekh, Chabaud, Kashefi

Figures of merit

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User/Device tunability resources





Enforcing the *correct functioning* of a quantum device, using the minimum amount of *resources*, while making as few *assumptions* as possible.

User/Device tunability resources Noise Model





Ongoing Project for Practical Implementation

Ongoing Project for Practical Implementation



Ongoing Project for Practical Implementation












1 TENNER

Standard





QUANTUM INTERNET ALLIANCE



