QSS56 - Paula Garcia Molina, Gadi Afek, Chi Shu - Questions & Answers

Paula Garcia Molina, Gadi Afek, Chi Shu

Table of Contents

1. Paula Garcia Molina

Can you give some insights on the whole computational complexity of the algorithm? Is it there some dependence of this complexity with the input for the PDE?

PAULA: The algorithm is composed of several operations that affect its complexity and cost. The quantum Fourier interpolation to go from a n-qubit solution to a (n+m)-qubit one uses n+m qubits and $O((n+m)^2)$ quantum gates, showing an advantage with respect to the classical one using the FFT with $O(2^{(n+m)})$ real values and $O((n+m)\ 2^{(n+m)})$ operations. Then, the cost is also affected by the ansatz. In the ZGR the number of parameters (and hence quantum gates) grows exponentially with the number of qubits, and we have observed that this is the most efficient ansatz. For the RY this scaling is linear. Finally, the classical optimizer also affects the cost, for example, SPSA makes 2 evaluations of the cost function (each with the corresponding number of shots) per iteration, while ADAM needs $2n_{\text{parameters}}+1$ evaluations of the cost function. When it comes to the dependence of the input PDE, the form of the solution affects this computational complexity, as those solutions that better meet the requirements (defined over a regular domain, periodic in that domain, bandwidth-limited, vanishes towards the boundaries of the interval) are expected to need a smaller number of qubits to reach a good approximation. This is because if they have these characteristics Fourier interpolation works as it best and we can benefit from the small error in the interpolation, which is $O(2^{(-np)})$ for solutions whose first p-1 derivatives are periodic, and doubly-exponential in the number of qubits for analytic functions, so the form of the solution affects the number of qubits necessary to reach a certain fidelity.

When you set up the variational ansatz, even after applying symmetries is there a trade-off between having more parameters (which can make the variational solution more accurate), and having fewer parameters (to make the classical optimisation and number of steps less costly)? In general, what is the best way to choose this?

PAULA: One has to find a balance between the number of parameters and the cost of the optimization. Theoretically, the more parameters the better the optimization, as we are supposed to have a greater flexibility to reproduce the solution. However, when it comes to the real application the presence of barren plateaus make it very difficult for the optimizer to find the minimum and it gets stuck at worse values of the parameters than for the ones corresponding to a lower number of qubits. So first, we have to be aware of this phenomenon and avoid values of qubits that lead to a number of parameters for which this happens. Once we have avoided it, we have to find the trade-off between the fidelity and errors that we want to obtain and the cost we are willing to have. Depending on the number of times you want to repeat each simulation and the resources you have you determine if it is worth the improve in the solution obtained from a greater number of parameters or if it is better to reduce the cost and obtain a solution that, while worse than the one for a greater number of qubits, it is still very good.

What is the likelihood of seeing a practical quantum advantage from these types of variational algorithm? And are they going to be limited by the quantum devices, or the classical optimisation?

PAULA: It depends on the concrete variational algorithm. While they have common elements, they have differences that modify this likelihood, so I am going to focus on our proposal. I think that for this concrete application is it complicated to see a practical quantum advantage, firstly due to the limitations of current quantum devices. In order to reach higher accuracies we need to increase the number of measurements in several orders of magnitude, we used 8192 shots and we are talking to use about 100 to 10000 times more measurements for solving a PDE such as the one of the transmon qubit. This is not possible in current quantum computers as the access and temporal stability is limited. In addition, if we consider the effect of noise we need a low number of qubits to obtain a considerably good result (low effect of noise), but for such low number of qubits we cannot always reach a sufficiently high fidelity. Therefore, quantum devices limit this due to the number of measurements and the noise, and these limitations affect in fact most variational algorithms. When it comes to the classical optimization it also limits the performance, as we have seen that depending on the optimizer it can be very costly. Moreover, the election of the ansatz also affects this due to barren plateaus.

Does this apply to 1D PDEs of any order? Can it be extended to 2D and 3D?

PAULA: Currently we have focused on second order PDEs, but we could adapt it to other orders. For extensions to higher dimensions we need to use a multidimensional encoding of the function in the quantum register and also consider the effect in the Fourier transform.

Can you say something about the influence of gate errors on the protocol?

PAULA: We have observed that noise affects our protocol, as it decreases the fidelity of the solution and increases the error in the estimation of the energy. This is caused by the gate errors and decoherence, and we analyzed their joint effect in the performance of the algorithm. For more qubits we have more quantum gates (as the number of gates of the quantum Fourier transform and the ansatz increases) and more error associated to them. This influences the protocol, making the fidelity decrease. However, for a sufficiently small number of gates and total duration of the circuit we can still obtain high fidelities, even though the error in the estimation of the energy is very significant. This is because the noise produces a shift in the cost function so we can still obtain a good approximation of the optimum parameters for this case.

2. Gadi Afek

Is it important that the sphere is really round, i.e. on what scale do irregularities matter?

GADI: Asphericity couples rotations to the center of mass motion adding noise to the measurement. Our spheres are typically spherical to within ~ 1% and we use the fact that we can spin them fast to decouple the rotation from the center of mass.

Even the particle is neutralized, it most likely contains permanent dipoles. Is it relevant? If so, how can you control this?

GADI: Correct. Even a discharged particle contains some "baked-in" charge asymmetry which is of the order of ~100-1000 *eu*m (depending on sphere size, material and manufacturing process). The most basic thing one can do to avoid dipole forces is to make the electric field as homogeneous as possible, because dipoles couple to gradients in the electric field. Second, the spinning allows averaging-out of any component of the dipole perpendicular to the spin axis. Finally, by controlled precessions of the sphere it is possible to align the dipole vector in space.

What are the ideas to cool these large spheres to the standard quantum limit?

GADI: Since at this point we are technical-noise-limited, improvements of our laser-pointing stability and vacuum pressure should bring us closer than the ~100 X SQL we are at now.

Once you overshoot to +ve charges when discharging the spheres, how do you continue the process so as to lead to zero charge?

GADI: At high field amplitudes (typically > 100 V/mm) electrons are ejected from the sphere. When we want to add electrons to the sphere we reduce the field amplitude to ~< 10 V/mm and are then able to eject electrons from neighboring surfaces, some of which get stuck in the sphere.

3. Chi Shu

What is the trade off in building clocks between the advantages of squeezing vs. a reduced probe time because of decoherence? What are the near-term opportunities for this to impact practical and/or high-precision clocks?

CHI: If a clock is limited by excited state lifetime τ, entanglement can't improve clock ultimate performance (with largest Ramsey time allowed) due to its higher sensitivity to decoherence. However, clocks with entangled states, such as spin squeezed states, can improve their performances for sensing signals at higher frequency than $1/\tau$. This quantum enhancement is particularly useful for sensing signals with AC components such as gravitational waves and dark matter related physics. Currently, most of the clocks are limited by the local oscillator laser's coherence time. In such a case, the quantum improvement of squeezing is always helpful as long as Dick noise is not significant.

Is time reversal QM limited to the symmetric states you can generate in one axis twisting or can this be generalized to entangled states with large quantum Fisher information? e.g. what if you would have short range interactions?

CHI: The time-reversal QM should be able to improve sensing for any state with large quantum Fisher information as long as the decoherence in the reversed evolution is not significant. Short range interactions could also be used in this particular case. It is possible to even sense non-local signals with time-reversed short range interactions. I would be happy to discuss more details in followup emails.

How does this method of preparing squeezing via unitary dynamics compare with methods based on performing a collective quantum projective measurement?

CHI: Measurement-based Squeezing via collective quantum projective measurement requires high quantum efficiency of detectors to achieve low broadening of the state, while the unitary dynamic squeezing doesn't have such a requirement. Generally speaking, any scattering or transmission of the light carries information of the atomic state. If one can measure all photons scattered, it is possible to achieve unitary operation even for measurement-based squeezing. However, that is practically hard. In our cavity feedback squeezing, we purposely have light away from vacuum Rabi splitting peaks to reduce the amount of atomic state information leaked into the environment, thus achieving a high unitarity of the evolution.